Currency Substitution and Monetary Policy under the Incomplete Financial Market

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ABSTRACT

Currency substitution is a phenomenon that domestic residents use a foreign currency as a medium of exchange. It is a common feature of many developing and transition countries. Many literatures concern the role of currency substitution on the effects of monetary policy and on the real exchange rate fluctuations. In this paper, we analyze the extent to which the degree of currency substitution influences on the effects of monetary policy. Especially, we will focus on the role of real exchange rate in the transmission of monetary policy. To attain this objective, we develop a small open economy general equilibrium model which has a standard New Keynesian framework. Our model also assumes incomplete financial markets in which risk sharing conditions and uncovered interest rate parity condition do not hold and net foreign assets play a role. We derive a tractable model in terms of the output gap, domestic inflation rate and the real exchange rate gap. Our impulse responses and unconditional variances analyses show that the degree of currency substitution does not have serious influences on the effects of domestic monetary policy. This is because the real interest rate channel dominates the marginal utility of consumption channel. On the other hand, it influences on the transmissions of foreign monetary policy shock to the domestic economy. Moreover, in the incomplete financial market, different degree of currency substitution has different permanent effects on the domestic economy through the net foreign assets.

Keywords: Currency Substitution, Small Open Economy DSGE Model, Money-in-the-Utility-Function Model, Incomplete Financial Market, Real Exchange Rate

JEL Classification: E50, E52, F32, F41

1. Introduction

Currency substitution is a phenomenon that domestic residents use a foreign currency as a medium of exchange[1]. It is a common feature of many developing and transition countries which had experienced especially high inflation in past times. In condition of high inflation, the domestic residents want to protect from high costs of using and holding domestic currency, and looks for alternatives, for example, the U.S. dollar.

There are many theoretical and empirical literatures concerning currency substitution[2]. These literatures are classified into two generous classes. One includes literatures which examine whether the phenomena of currency substitution can be detected in particular countries. The other includes literatures which examine the effects of currency substitution on the economy, especially examine following three main questions.

The first question concerns the role of currency substitution on the effects of monetary policy. Végh (2013) points out two channels through which currency substitution has effects on monetary policy independence. First, a high degree of currency substitution would make the nominal interest rate react strongly to even small changes in monetary policy, which makes nominal exchange rate more volatile. Kareken and Wallace (1981) argue that the nominal exchange rate becomes indeterminate when domestic and foreign currencies are perfect substitute. Needless to say, an introduction of tiny imperfect substitution is enough to make the nominal exchange rate determinate, however, their result highlights the potential instabilities caused by currency substitution. Second, monetary policy has effects on trade balance, which has also effects on consumption. Under flexible exchange rates without currency substitution, for example, a reduction in the rate of money growth leads to an immediate reduction in the inflation rate without any real effects. In the presence of currency substitution, however, the same reduction leads to an increase in demand for foreign currency. When foreign currency is the only internationally traded asset, a fall in consumption is needed to run a trade surplus so as to re-adjust foreign money balances. In that sense, the presence of currency substitution would undermine the effectiveness of monetary policy. However, the second channel does not matter, when domestic households can hold not only foreign currency but also foreign bond. This is because the increase in the money demand for foreign currency is offset by the same decline in the holdings of foreign bond in equilibrium, which does not influence on the accumulation of net foreign assets; therefore, on trade balance. Another channel through which currency substitution influences on monetary policy independence is a degree of insulation from foreign shocks. Rogers (1990) analyzes the transmission effects of permanent rise in foreign inflation under flexible exchange rates and currency substitution, and shows that a degree of insulation from foreign price shocks may be lost in the present of currency substitution. Felices and Tuesta (2007) and Batini, Levine and Pearlman (2008) construct a Dynamic Stochastic General Equilibrium (DSGE) model of currency substitution. They show that marginal utility of consumption depends on weighted average of home and foreign nominal interest rates, in which the weights are determined by the degree of currency substitution. Those features imply that as the degree of currency substitution becomes higher, the ability of the central bank to stabilize inflation rate and the output using domestic nominal interest rate diminishes.

The second question concerns the effects of currency substitution on real exchange rate fluctuations in a flexible exchange rate regime. There has been a controversy over the answer to this question. Calvo and Rodriguez (1977) construct a no-bond economy model with tradable and non-tradable goods to show that an increase in the rate of money growth leads to a real depreciation. The increase in the rate of money growth implies that consumers wish to increase the ratio of foreign to domestic currency. Since foreign currency is the only internationally traded asset in their model, the economy must run a trade surplus to increase its holdings of foreign currency. Hence, the equilibrium response to trade surplus is a depreciation of the real exchange rate. On the other hand, Liviatan (1981) uses a similar model but shows that an increase in the rate of money growth leads to an appreciation of the real exchange rate. The

main differences between their models are in the specification of money demand function. Calvo and Rodriguez (1977) assume that there is a substitution between home and foreign currencies. On the other hand, Liviatan (1981) assumes that domestic and foreign currencies are held in fixed proportions of "composite money", which is defined as a composite of domestic and foreign currencies. Therefore, in Liviatan (1981), the rise in the rate of money growth, and hence inflation, reduces the demand for this composite money. The only way for the economy to get rid of foreign money balances is to run a trade deficit, which requires an appreciation of the real exchange rate. Végh (2013) puts this controversy in order and point out that whether an increase in the rate of money growth leads to a real appreciation or depreciation depends on the relative size of the elasticity of substitution between the two currencies and the elasticity of substitution between consumption and liquidity services. If the former is smaller (larger) than the latter, then Liviatan (1981) (Calvo & Rodriguez, 1977) case appears so that an increase in money growth leads to a real appreciation (depreciation). However, as mentioned above, these discussions do not matter, when foreign bond is avilable. The other important channel through which currency substitution influences on the real exchange rate is a risk sharing condition between domestic and foreign countries. At this point, we will refer later.

Last question concerns the effects of currency substitution on seigniorage and/or inflation tax. The higher the substitutability of domestic and foreign currency is, the more difficult it is for the government to finance deficits by printing money. On the one hand, seigniorage is taken up by the foreign money holdings, and the demand for domestic currency would become more sensitive to the seigniorage rate. Hence, seigniorage revenues would be lower in the presence of currency substitution. Calvo and Végh (1992) show that the optimal inflation rate which maximizes seigniorage is lowered in the presence of currency substitution.

The aim of our paper is related to first two questions. In this paper, we will analyze the extent to which the degree of currency substitution influences on the effects of monetary policy in the DSGE framework. Our paper is closely related to Felices and Tuesta (2007) and Batini, Levine and Pearlman (2008), in which monetary policy under the currency substitution is discussed in the DSGE framework. But, there are some departures from these literatures. Especially, we will focus on the role of real exchange rate in the transmission of monetary policy. We will derive a tractable model in terms of the output gap, domestic inflation rate and deviations of the real exchange rate from its natural level (hereafter, the real exchange rate gap). To attain this objective, our model assumes incomplete financial markets in which agents cannot access to complete sets of state contingent bonds. In the complete financial market, risk sharing conditions across countries and uncovered interest rate parity condition always hold. However, Selaive and Tuesta (2003) mention that "two important issues in international macroeconomics are the apparent lack of risk sharing across countries and the uncovered interest rate parity failure". The risk sharing condition states that the real exchange rate evolves according to the marginal rate of substitution between domestic and foreign countries; as a result, it predicts a high cross-correlation between the real exchange rate and relative consumption. However, Chari, Kehoe and McGrattan (2002) find that the data does not show a clear pattern, and they refer to this discrepancy as *the consumption-real exchange rate anomaly*. The uncovered interest rate parity condition states that the expected change in the nominal exchange rate is proportional to the interest rate differential. However, there are vast empirical evidences suggesting that it does not hold. For example, Lane and Milesi-Ferretti (2001) focus a significant role of the net foreign assets in determining the real interest rate differential. In contrast, in the incomplete financial market, the uncovered interest parity condition does not hold, and the deviation from the uncovered interest rate is affected by net foreign assets. Thus, the existence of net foreign assets breaks the tight link between real exchange rate and relative consumption.

The remainder of this paper is organized as follows. In Section 2, we will introduce our small open economy DSGE model, which has a standard New Keynesian framework with

monopolistically competitive market structure and staggered price setting. We introduce currency substitution into our model through the money-in-the-utility-function framework under the incomplete financial market structure. In Section 3, we will present quantitative results based on calibration analyses of our model. Section 4 is a conclusion.

2 The Model

Our model is based on a DSGE model for a two country open economy, which has a standard New Keynesian framework with monopolistically competitive market structure and staggered price setting. We also introduce currency substitution into our model through the money-in-the-utility-function framework. We also assume an incomplete financial market structure.

2.1 Households

There are two countries, called home country and foreign country, respectively. The world size is normalized to unity. The population in the home country lies on the interval $[0,n]$, while in the foreign economy it lies on $(n,1]$. We allow for tradable goods only. Differentiated goods are produced by a continuum of monopolistic competitive firms. Goods indexed by $[0,n]$ are produced in home country (goods H), while goods indexed by $(n,1]$ are produced in foreign country (goods F). Both generic agent h belonging to the home economy and an agent f belonging to the foreign economy consumes both goods H and F. We give money a role as medium of exchange through a money-in-the-utility-function framework. An agent h holds not only home currency (currency H) but also foreign currency (currency F) to obtain liquidity service. Its utility depends positively on consumption and real holdings of currency H and F, while negatively on labor supply. Therefore, at time $t=0$, an agent h maximizes the following expected value of a discounted stream of period utility,

$$E_0\sum_{t=0}^{\infty}\beta^t U^h\left(C_t,\frac{M_{H,t}^d}{P_t},\frac{S_t M_{F,t}^d}{P_t},N_t^s\right) \tag{1}$$

where

$$U^h\left(C_t,\frac{M_{H,t}^d}{P_t},\frac{S_t M_{F,t}^d}{P_t},N_t^s\right)=\frac{X_t^{1-\sigma}}{1-\sigma}-\frac{\left(N_t^s\right)^{1+\varphi}}{1+\varphi}$$

$$X_t=\left[\omega C_t^{\frac{\theta-1}{\theta}}+(1-\omega)Z_t^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

$$Z_t=\left[\gamma\left(\frac{M_{H,t}^d}{P_t}\right)^{\frac{\nu-1}{\nu}}+(1-\gamma)\left(\frac{S_t M_{F,t}^d}{P_t}\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}.$$

β is a discount factor, $M_{H,t}^d$ and $M_{F,t}^d$ are nominal holdings of currency H and F in terms of each country's currency at the end of period t, respectively. P_t is a consumer price index (defined below), S_t is an nominal exchange rate defined by the price of foreign currency in terms of home currency and N_t^s is total hours of labor supply. X_t is a consumption-currency index, ω is the weight of consumption in the consumption-currency index, Z_t is a currency index, σ is a coefficient of risk aversion, φ is an inverse of elasticity of labor supply, θ is an elasticity of substitution between consumption and currency index, γ is a weight on currency H in the currency index and ν is an elasticity of currency substitution between currency H and F. C_t is a consumption index defined by familiar Dixit-Stiglitz form,

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{2}$$

$C_{H,t}$ and $C_{F,t}$ denote consumption sub-indices of goods *H* and *F*, respectively, and $\eta > 0$ is an elasticity of substitution between goods *H* and *F*. $0 < \alpha < 1$ measures an import share which represents its preference for foreign goods. Following Sutherland (2005), we assume that α is a function of the relative size of the foreign economy $1-n$ and of the degree of openness a, so that $\alpha = (1-n)a$. The consumption sub-indices $C_{H,t}$ and $C_{F,t}$ are defined as follows,

$$C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \; C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3}$$

$C_{H,t}(j)$ for $j \in [0,n]$ and $C_{F,t}(j)$ for $j \in (n,1]$ denote a consumption of differentiated good *j*, and $\varepsilon > 1$ is an elasticity of substitution among differentiated goods.

The agent *h* faces the following inter-temporal budget constraint,

$$\begin{aligned} &\int_0^n P_{H,t}(j)C_{H,t}(j)dj + \int_n^1 P_{F,t}(j)C_{F,t}(j)dj + M_{H,t} + S_t M_{F,t} + P_{BH,t}B_{H,t} + S_t P_{BF,t}B_{F,t} \\ &= W_t N_t^s + M_{H,t-1} + S_t M_{F,t-1} + B_{H,t-1} + S_t B_{F,t-1} + \Gamma_{H,t} + T_{H,t} \end{aligned} \tag{4}$$

where $P_{H,t}(j)$ for $j \in [0,n]$ and $P_{F,t}(j)$ for $j \in (n,1]$ denote prices of differentiated good *j* in terms of currency *H*, W_t is a nominal wage rate, $\Gamma_{H,t}$ are dividends from ownership of firms, and $T_{H,t}$ are exogenous nominal lump-sum cash transfers. There are two risk-free one-period bonds; bond *H* and bond *F*. They pay one unit of currency *H* and *F*, respectively, and are denominated in each country's currency. We denote the agent *h*'s holdings of bond *H* and *F* at the end of period *t* as $B_{H,t}$ and $B_{F,t}$. $P_{BH,t}$ and $P_{BF,t}$ are the prices of these bonds, respectively. Here, we assume that the agent *h* can hold both bonds, while the agent *f* can hold only bond *F*. Furthermore, we assume that when the agent *h* holds bond *F*, it faces transaction cost in the form of risk premium[3]. The prices of these bonds are given by

$$P_{BH,t} = \frac{1}{1+i_{H,t}}, \; P_{BF,t} = \frac{1}{1+i_{F,t}}. \tag{5}$$

$i_{H,t}$ and $i_{F,t}$ denote risk-free one period nominal interest rates of these bonds, which are applied for the agent *h*. When we denote risk-free one period nominal interest rates of bond *F* applied for the agent *f* as $i_{F,t}^*$, then a following condition is met,

$$1 + i_{F,t} = (1 + i_{F,t}^*)\Psi\left(\frac{S_t(B_{F,t} + M_{F,t})}{P_H Y}\right) \tag{6}$$

where

$$\Psi\left(\frac{S_t(B_{F,t} + M_{F,t})}{P_H Y}\right) = -\kappa\left\{\exp\left(\frac{S_t(B_{F,t} + (M_{F,t} - M_F))}{P_H Y}\right) - 1\right\}. \tag{7}$$

$\Psi(\cdot)$ captures the transaction cost which satisfies $\Psi(0) = 1$ and $\Psi'(\cdot) < 0$. P_H is a price sub-index of goods *H* (derived below) and Y is an aggregate output of goods *H*. Variables without time subscript denote its initial steady state value. Equation (6) means that the transaction cost depends on the ratio of economy-wide holdings of net foreign assets ($S_t(B_{F,t} + M_{F,t})$) to the initial steady state level of nominal output ($P_H Y$). This ratio is taken as given by the agent *h*. Equation (7) means that at the initial steady state in which $B_{F,t} = 0$ and $M_{F,t} = M_F$, then $\Psi(0) = 1$. This specification is consistent with Lane and Milesi-Ferretti (2001) in which they

assign a significant role of the net foreign assets in determining the real interest rate differential.

Optimal allocations of expenditure within differentiated goods are given by

$$C_{H,t}(j)=\frac{1}{n}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t}\,,\ C_{F,t}(j)=\frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}\,. \tag{8}$$

$P_{H,t}$ and $P_{F,t}$ are price sub-indices of goods H and F defined as

$$P_{H,t}=\left[\left(\frac{1}{n}\right)\int_0^n P_{H,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}},\ P_{F,t}=\left[\left(\frac{1}{1-n}\right)\int_n^1 P_{F,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}. \tag{9}$$

Similarly, optimal allocations of expenditure between goods H and F are given by

$$C_{H,t}=(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t\,,\ C_{F,t}=\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t\,. \tag{10}$$

P_t is a consumer price index defined as

$$P_t=[(1-\alpha)(P_{H,t})^{1-\eta}+\alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}\,. \tag{11}$$

Note that combining the optimality conditions (equations (8) and (10)) with the definitions of consumption indices (equations (2) and (3)) and the price indices (equations (9) and (11)) yields

$$\int_0^n P_{H,t}(j)C_{H,t}(j)dj=P_{H,t}C_{H,t}\,,\ \int_n^1 P_{F,t}(j)C_{F,t}(j)dj=P_{F,t}C_{F,t}\,,$$

$$P_{H,t}C_{H,t}+P_{F,t}C_{F,t}=P_tC_t\,.$$

Therefore, we can rewrite the intertemporal budget constraint (4) as

$$\begin{aligned}&P_tC_t+M_{H,t}+S_tM_{F,t}+P_{BH,t}B_{H,t}+S_tP_{BF,t}B_{F,t}\\&\quad=W_tN_t^S+M_{H,t-1}+S_tM_{F,t-1}+B_{H,t-1}+S_tB_{F,t-1}+\Gamma_{H,t}+T_{H,t}.\end{aligned} \tag{12}$$

First order conditions are given by (we omit the superscript h)

$$\frac{U_{N^S,t}}{U_{C,t}}=\frac{W_t}{P_t} \tag{13}$$

$$U_{C,t}=\beta(1+i_{H,t})E_t\left[\frac{P_t}{P_{t+1}}U_{C,t+1}\right] \tag{14}$$

$$U_{C,t}=\beta(1+i_{F,t})E_t\left[\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}U_{C,t+1}\right] \tag{15}$$

$$\frac{U_{C,t}}{P_t}=\frac{U_{M_H^d/P,t}}{P_t}+\beta E_t\left[\frac{U_{C,t+1}}{P_{t+1}}\right] \tag{16}$$

$$\frac{S_tU_{C,t}}{P_t}=\frac{S_tU_{SM_F^d/P,t}}{P_t}+\beta E_t\left[\frac{S_{t+1}U_{C,t+1}}{P_{t+1}}\right] \tag{17}$$

where marginal utilities can be calculated as

$$U_{C,t}=X_t^{\frac{1}{\theta}-\sigma}C_t^{-\frac{1}{\theta}}\omega$$

$$U_{\frac{M_H^d}{P},t}=X_t^{\frac{1}{\theta}-\sigma}(1-\omega)\gamma Z_t^{\frac{1}{\nu}-\frac{1}{\theta}}\left(\frac{M_{H,t}^d}{P_t}\right)^{-\frac{1}{\nu}}$$

$$U_{\frac{SM_F^d}{P},t}=X_t^{\frac{1}{\theta}-\sigma}(1-\omega)(1-\gamma)Z_t^{\frac{1}{\nu}-\frac{1}{\theta}}\left(\frac{S_tM_{F,t}^d}{P_t}\right)^{-\frac{1}{\nu}}$$

$$U_{N^s,t} = N_t^{s,\varphi}.$$

Equation (13) is an intratemporal optimal condition, which relates the marginal rate of substitution between labor and consumption to the real wage (labor supply schedule). Equations (14) and (15) are intertemporal Euler equations for consumption. Equation (16) states that the marginal utility of consumption at time t equals to the marginal benefit of adding one unit of currency H to money holdings at time t. The latter has two components, namely, the marginal utility that the household can obtain from holding one unit of currency H and from consumption at time $t+1$ with the extra currency unit. Equation (17) is a counterpart for currency F.

As discussed in Galí (2008), the marginal utility of consumption $U_{C,t}$ means that in particular case in which the intertemporal and the intratemporal elasticities of substitution coincide, namely $1/\theta = \sigma$, optimality conditions are the same as in the case of separable utility function. In this case, money becomes neutral. However, in the general case in which $1/\theta \neq \sigma$, both labor supply and Euler equation are influenced by the currency index Z_t through the dependence on the consumption-currency index X_t. Moreover, as shown below, the demand for Z_t depends on not only home nominal interest rate i_H but also foreign nominal interest rate i_F. Those features imply that different monetary policies in both countries cause different paths of i_H, i_F ($i_{F,t}^*$) and therefore of Z_t. Then, different level of Z_t has different implications for $U_{C,t}$, therefore, on the position of the labor supply schedule, employment and output. Especially, important feature is that $U_{CZ,t} > 0$ iff $1/\theta > \sigma$, in which case consumption and the currency index are *complements*. In this case, a rise in home nominal interest rate leads to a decline in Z_t, which decreases $U_{C,t}$. On the other hand, $U_{CZ,t} < 0$ iff $1/\theta < \sigma$, in which case consumption and the currency index are *substitutes*.

Combining equation (14) with (16) yields the demand function for currency H,

$$\frac{U_{M_H^d/P,t}}{U_{C,t}} = \frac{i_{H,t}}{1+i_{H,t}}. \tag{18}$$

Similarly, combining equation (15) with (17) yields the demand function for currency F,

$$\frac{U_{SM_F^d/P,t}}{U_{C,t}} = \frac{i_{F,t}}{1+i_{F,t}}. \tag{19}$$

Using equations (18) and (19) and the specification of utility function, we obtain the relative money demand function for currency F with respect to currency H as,

$$RM_t^d \equiv \frac{S_t M_{F,t}^d / P_t}{M_{H,t}^d / P_t} = \left(\frac{\gamma}{1-\gamma} \frac{1+i_{H,t}}{i_{H,t}} \frac{i_{F,t}}{1+i_{F,t}} \right)^{-\nu}. \tag{20}$$

From equation (20), we can see that the relative money demand changes endogenously with monetary policies in both countries which cause the change in the nominal interest rates $i_{H,t}$ and i_F ($i_{F,t}^*$) and nominal exchange rate S_t. Since $dRM_t^d / di_{H,t} > 0$, an increase in the opportunity cost of holding currency H, namely $i_{H,t}$, increases the ratio of currency F to H. It means that high inflation country, in which the nominal interest rate might also be higher, would have a higher ratio of foreign currency.

As for the foreign economy, we follow Felices and Tuesta (2007) to assume that the generic agent f obtains utility only from consumption and labor supply. Thus, it maximizes the following expected value of a discounted stream of period utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U^f \left(C_t^*, N_t^{*s} \right) \tag{21}$$

where

$$U^f\left(C_t^*, N_t^{*s}\right) = \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{*s,1+\varphi}}{1+\varphi}.$$

N_t^{*s} is total hours of labor supply. C_t^* is a consumption index defined as

$$C_t^* = \left[(1-\alpha^*)^{\frac{1}{\eta}}(C_{H,t}^*)^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}}(C_{F,t}^*)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{22}$$

where we assume that $1-\alpha^* = na$ as in Sutherland (2005). $C_{H,t}^*$ and $C_{F,t}^*$ are consumption sub-indices defined as

$$C_{H,t}^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}}\int_0^n C_{H,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}},\ C_{F,t}^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}}\int_n^1 C_{F,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{23}$$

$C_{H,t}^*(j)$ for $j \in [0,n]$ and $C_{F,t}^*(j)$ for $j \in (n,1]$ denote the agent *f*'s consumption of differentiated good *j*.

Optimal allocations of expenditure within differentiated goods are given by

$$C_{H,t}^*(j) = \frac{1}{n}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t},\ C_{F,t}^*(j) = \frac{1}{1-n}\left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t} \tag{24}$$

where $P_{H,t}^*(j)$ for $j \in [0,n]$ and $P_{F,t}^*(j)$ for $j \in (n,1]$ are prices of good *j* in terms of currency *F*. $P_{H,t}^*$ and $P_{F,t}^*$ are price sub-indices of goods *H* and *F* in terms of currency *F* defined as

$$P_{H,t}^* = \left[\left(\frac{1}{n}\right)\int_0^n P_{H,t}^*(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}},\ P_{F,t}^* = \left[\left(\frac{1}{1-n}\right)\int_n^1 P_{F,t}^*(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \tag{25}$$

Optimal allocations of expenditure between goods *H* and *F* is given by

$$C_{H,t}^* = (1-\alpha^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*,\ C_{F,t}^* = \alpha^*\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* \tag{26}$$

where P_t^* is the foreign consumer price index defined as

$$P_t^* = [(1-\alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^*(P_{F,t}^*)^{1-\eta}]^{\frac{1}{1-\eta}}. \tag{27}$$

An agent *f* faces an intertemporal budget constraint given by[4]

$$P_t^* C_t^* + M_{F,t}^* + P_{BF,t}^* B_{F,t}^* = W_t^* N_t^{*s} + M_{F,t-1}^* + B_{F,t-1}^* + \Gamma_{F,t}^* + T_{F,t}^* \tag{28}$$

where $P_{BF,t}^*$ is the price of bond *F*, which is applied for the agent *f*,

$$P_{BF,t}^* = \frac{1}{1+i_{F,t}^*}. \tag{29}$$

First order conditions are given by (we omit the superscript *f*)

$$\frac{U_{N^{*s},t}}{U_{C^*,t}} = \frac{W_t^*}{P_t^*} \tag{30}$$

$$\frac{U_{C^*,t}}{P_t^*} = \beta(1+i_{F,t}^*)E_t\left[\frac{U_{C^*,t+1}}{P_{t+1}^*}\right] \tag{31}$$

where marginal utilities can be calculated as

$$U_{C^*,t} = C_t^{*-\sigma}$$

$$U_{N^{*s},t} = N_t^{*s,\varphi}.$$

2.2 Terms of trade and the real exchange rate

Here, we derive some relationships among the terms of trade, the real exchange rate and prices of goods. We define the terms of trade as

$$\mathrm{T}_t \equiv \frac{P_{H,t}}{P_{F,t}}. \tag{32}$$

We assume full pass-through of the exchange rate to the import price. Therefore, the law of one prices holds,

$$P_{H,t}(j) = S_t P_{H,t}^*(j),\ P_{F,t}(j) = S_t P_{F,t}^*(j). \tag{33}$$

Equation (33) implies that the law of one prices also hold for aggregate goods H and F,

$$P_{H,t} = S_t P_{H,t}^*,\ P_{F,t} = S_t P_{F,t}^*. \tag{34}$$

Using equations (11), (27) and (34), we can express the real exchange rate as

$$\begin{aligned} Q_t &= \frac{S_t P_t^*}{P_t} = \frac{\left[(1-\alpha^*)(S_t P_{H,t}^*)^{1-\eta} + \alpha^*(S_t P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}} \\ &= \left[\frac{(1-\alpha^*)\mathrm{T}_t^{1-\eta} + \alpha^*}{(1-\alpha)\mathrm{T}_t^{1-\eta} + \alpha}\right]^{\frac{1}{1-\eta}} = \left[\frac{1}{(1-a)\mathrm{T}_t^{1-\eta} + a}\right]^{\frac{1}{1-\eta}},\ as\ n \to 0 \end{aligned} \tag{35}$$

where the last equality is obtained by taking the limit of n as $n \to 0$, which corresponds to the small open economy assumption. Equation (35) says that even if the law of one prices holds, the purchasing power parity condition does not hold, since the weights assigned to goods H and F in consumption index are asymmetric between home and foreign countries.

2.3 Firms

We assume that there exists a continuum of monopolistic competitive firms, and a firm indexed by j produces differentiated good j. Goods indexed by $j \in [0,n]$ are produced in home country, while goods $j \in (n,1]$ are produced in foreign country. All domestic firms use an identical technology represented by a constant return to scale production function,

$$Y_{H,t}(j) = A_t N_t(j),\ j = [0,n] \tag{36}$$

where $N_t(j)$ is labor input of firm j. A_t represents productivity shock which is common to goods H sector, and evolves exogenously according to the following law of motion,

$$\frac{A_t}{A} = \left(\frac{A_{t-1}}{A}\right)^{\rho_A} \exp[\varepsilon_{A,t}] \tag{37}$$

where $0 \le \rho_A \le 1$ is an autoregressive coefficient and $\varepsilon_{A,t}$ is a random variable serially uncorrelated and normally distributed, with zero mean and constant variance σ_A^2.

The firm's cost minimization problem is specified as following Lagrangian equation,

$$L_t = \frac{W_t}{P_{H,t}} N_t(j) + \Phi_t(j)(A_t N_t(j) - Y_t(j)). \tag{38}$$

From the first order condition, we can see that real marginal cost equals across firms and equal to Lagrangian multiplier,

$$\frac{W_t / P_{H,t}}{A_{H,t}} = \Phi_t(j) = \Phi_t. \tag{39}$$

Following Calvo (1983), we assume that a fraction of $1-\chi$ of firms can set a new price in each period. In other words, each firm is able to set a new price with probability $1-\chi$ in each period. With probability χ, it cannot reset price so that its price is just the average price of all firms that prevailed at time t-1, $P_{H,t-1}$. While individual firms produce differentiated goods, they all have the same production function and face the demand function with constant and the same elasticities. Thus, all price-resetting firms set the same prices. Let $P^{+}_{H,t}$ denote newly price set optimally (we omit the subscript *j*), then, the price sub-index of goods *H* (9) can be written as

$$P_{H,t} = \left[\chi P^{1-\eta}_{H,t-1} + (1-\chi)P^{+,1-\eta}_{H,t}\right]^{\frac{1}{1-\eta}}. \tag{40}$$

Since production function are homogeneous so that average cost equals to marginal cost, present discounted value of firms' profit can be written as

$$\max_{\{P_{H,t}\}} E_t\left[\sum_{k=0}^{\infty}\chi^k \Xi_{t,t+k}\left\{\left(P^{+}_{H,t} - P_{H,t+k}\Phi_{t+k}\right)Y_{t,t+k}\right\}\right] \tag{41}$$

where $\Xi_{t,t+k} \equiv \beta^k (U_{C,t+k}/U_{C,t})(P_t/P_{t+k})$ is a stochastic discount factor, and $Y_{t,t+k}$ is an output at period $t+k$ for a firm that last reset its price at period t.

The market clearing condition of goods *j* for $j=[0,n]$ requires

$$Y_t(j) = nC_{H,t}(j) + (1-n)C^{*}_{H,t}(j). \tag{42}$$

From equations (8), (10), (33) and (34), equation (42) can be rewritten as

$$\begin{aligned} Y_t(j) &= (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{(1-\alpha^{*})(1-n)}{n}\left(\frac{P^{*}_{H,t}(j)}{P^{*}_{H,t}}\right)^{-\varepsilon}\left(\frac{P^{*}_{H,t}}{P^{*}_t}\right)^{-\eta} C^{*}_t \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \frac{(1-\alpha^{*})(1-n)}{n}C^{*}_t Q^{\eta}_t\right] \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left\{(1-a)C_t + aC^{*}_t Q^{\eta}_t\right\},\ as\ \ n\to 0 \end{aligned} \tag{43}$$

where the last equality is obtained by taking the limit of n as $n\to 0$, then $\alpha \to 1-a$ and $(1-\alpha^{*})(1-n)/n \to a$. Therefore, we can rewrite equation (41) as

$$\begin{aligned} \max_{\{P^{+}_{H,t}\}} E_t\Bigg[\sum_{k=0}^{\infty}\chi^k \Xi_{t,t+k}\Bigg\{ & P^{+}_{H,t}\left(\frac{P^{+}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\left(\frac{P_{H,t+k}}{P_{t+k}}\right)^{-\eta} - P_{H,t+k}\Phi_{t+k}\left(\frac{P^{+}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\left(\frac{P_{H,t+k}}{P_{t+k}}\right)^{-\eta}\Bigg\} \\ & \times\left\{(1-a)C_{t+k} + aC^{*}_{t+k}Q^{\eta}_{t+k}\right\}\Bigg]. \end{aligned}$$

Hence, first order condition for profit maximization is given by

$$E_t\left[\sum_{k=0}^{\infty}\chi^k \Xi_{t,t+k}Y_{t,t+k}\left(P^{+}_{H,t} - \frac{\varepsilon}{\varepsilon-1}P_{H,t+k}\Phi_{t+k}\right)\right] = 0 \tag{44}$$

where $\varepsilon/(\varepsilon-1)$ can be interpreted as a mark-up rate.

2.4 Government

We assume that the government balances its budget each period, which is not restrictive because Ricardian equivalence holds here. Moreover, without loss of generality, we assume that home government expenditures are zero. Therefore, we specify the government's budget constraint as

$$T_{H,t} = M^{s}_{H,t} - M^{s}_{H,t-1}. \tag{45}$$

$M^{s}_{H,t}$ denotes nominal money supply of currency *H* per capita. Equation (45) means that the money supply, which amounts to seigniorage, is injected through the form of lump sum

transfers. As for the foreign government's budget constraint, we assume that seigniorage revenues from foreign households are returned back to them through the form of lump sum transfers, while seigniorage revenues from home households are used to finance the government expenditures. For simplicity, we assume that foreign government expenditures do not have a direct effect on foreign household's utility and on the allocation of resources.

$$T_{F,t}^{*} = M_{F,t}^{*s,F} - M_{F,t-1}^{*s,F} \tag{46}$$

$$G_{t}^{*} = n(M_{F,t}^{s,H} - M_{F,t-1}^{s,H}) \tag{47}$$

where $M_{F,t}^{s,H}$ and $M_{F,t}^{*s,F}$ denote the amounts of currency F per capita circulated in home and foreign countries, respectively. G_t^* denotes total foreign government expenditures. Thus, if we denote total nominal money supply of currency F as $M_{F,t}^{*s}$, the following condition holds,

$$M_{F,t}^{*s} = nM_{F,t}^{s,H} + (1-n)M_{F,t}^{*s,F} . \tag{48}$$

2.5 Risk-sharing and uncovered interest rate parity

Under the assumption of incomplete international financial markets, risk sharing condition, which relates marginal elasticity of substitution between home and foreign household to the real exchange rate, only holds in expected first difference,

$$\Psi\left(\frac{S_t(B_{F,t}+M_{F,t})}{P_H Y}\right)E_t\left[\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\frac{U_{C,t+1}}{U_{C,t}}\right] = E_t\left[\frac{P_t^*}{P_{t+1}^*}\frac{U_{C^*,t+1}}{U_{C^*,t}}\right]. \tag{49}$$

From equations (6), (14) and (15), *modified* uncovered interest parity condition follows,

$$\frac{1+i_{H,t}}{(1+i_{F,t}^*)\Psi\left(\frac{S_t(B_{F,t}+M_{F,t})}{P_H Y}\right)} = \frac{E_t\left[(P_t/P_{t+1})(S_{t+1}/S_t)(U_{C,t+1}/U_{C,t})\right]}{E_t\left[(P_t/P_{t+1})(U_{C,t+1}/U_{C,t})\right]}. \tag{50}$$

From equations (49) and (50), we can see that net foreign assets would create deviation from uncovered interest parity, which breaks the tight link between the marginal elasticity of substitution and the real exchange rate.

2.6 Monetary policy rule

We specify the monetary policy rule within a family of Taylor rules, where the central bank responds to the natural real interest rate, output gap, inflation and the real exchange rate gap,

$$\frac{1+i_{H,t}}{1+i_H} = \frac{1+r_t^n}{1+r^n}\left(\frac{\Pi_{H,t}}{\Pi_H}\right)^{\psi_\pi}\left(\frac{Y_t}{Y_t^n}\right)^{\psi_x}\left(\frac{Q_t}{Q_t^n}\right)^{\psi_q}\exp[\nu_{iH,t}] \tag{51}$$

$$\nu_{iH,t} = \rho_{iH}\nu_{iH,t-1} + \varepsilon_{iH,t} \tag{52}$$

where $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ is gross inflation rate of home goods H. r_t^n is an unobservable natural real interest rate. Y_t^n and Q_t^n are natural level of output and the real exchange rate which prevail under the frictionless economy, where financial market is complete, price is flexible price and money is neutral. $\nu_{iH,t}$ is an exogenous component, which follows an AR(1) process. $\varepsilon_{iH,t}$ is a random variable serially uncorrelated and normally distributed, with zero mean and constant variance σ_{iH}^2. As discussed in Malik (2005), the parameter value of ψ_q implies the type of exchange rate regime. For example, $\psi_q = 0$ means the flexible exchange rate regime in which the central bank does not care about deviations of the real exchange rate from the target. $\psi_q > 0$ means the managed exchange rate regime in which the central bank responds to the movement of the real exchange rate by changing nominal interest rate. $\psi_q \to \infty$ means the fixed exchange rate regime.

2.7 Equilibrium

At first, we define aggregate output indices of home and foreign countries as

$$Y_t = \left[\left(\frac{1}{n}\right)\int_0^n Y_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},\ Y_t^* = \left[\left(\frac{1}{1-n}\right)\int_n^1 Y_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{53}$$

We also define an aggregate labor index so as to be consistent with equation (53),

$$N_t = \left[\left(\frac{1}{n}\right)\int_0^n N_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{54}$$

With these definitions, aggregate production function can be written as

$$Y_t = A_t N_t. \tag{55}$$

Substituting market clearing condition of good j for $j \in [0,n]$ (43) into equation (53) yields aggregate market clearing condition of goods H,

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left\{(1-a)C_t + aC_t^* Q_t^{\eta}\right\}. \tag{56}$$

Similarly, market clearing condition of good j for $j \in (n,1]$ requires

$$Y_t^*(j) = nC_{F,t}(j) + (1-n)C_{F,t}^*(j). \tag{57}$$

From equations (8), (10), (24) and (26), equation (57) can be rewritten as

$$\begin{aligned} Y_t^*(j) &= \frac{\alpha n}{1-n}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P_{F,t}^*(j)}{P_{F,t}^*}\right)^{-\varepsilon}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* \\ &= \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*}\right)^{-\varepsilon}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*\ ,as\ \ n \to 0 \end{aligned} \tag{58}$$

where the second equality follows from taking the limit of n as $n \to 0$, then $\alpha n/(1-n) \to 0$, $\alpha^* \to 1$. From equations (53) and (58), aggregate market clearing condition of goods F is

$$Y_t^* = \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{59}$$

As for the labor market, the clearing condition requires

$$N_t^s = N_t^d = N_t, \tag{60}$$

and

$$\frac{W_t}{P_t} = \frac{U_{N^s,t}}{U_{C,t}} = \frac{P_{H,t}}{P_t} A_t \Phi_t. \tag{61}$$

As for the money market of currency H, given nominal interest rates $i_{H,t}$ and $i_{F,t}^*$ ($i_{F,t}$) which are determined by monetary policy rules, the supply of currency H is fixed by the government to accommodate money demand for currency H in equation (18),

$$M_{H,t}^s = M_{H,t}^d = M_{H,t}. \tag{62}$$

Therefore, money market clearing condition of currency H does not influence the economy. As for the money market of currency F, the demand for currency F is determined by equation (19), while the demand for net foreign assets (namely, the amount of savings) is determined by equation (64) (shown below). It means that when the household wishes to change its holdings of currency F following the change of monetary policies, but does not wish to change its overall net foreign assets. For example, when it decreases its holdings of currency F, it simply uses

currency F to buy foreign bond. Therefore, money market clearing condition of currency F does not also influence the economy.

As for the bond market, net holdings of bond H are zero, $B_{H,t} = 0$, since home bonds are not held by foreign households. On the other hand, market clearing condition of bond F requires

$$nB_{F,t} + (1-n)B^*_{F,t} = 0 \tag{63}$$

where $B^*_{F,t}$ denotes the holdings of bond F by foreign household per capita.

Lastly, the accumulation of net foreign assets (current account) is determined by intertemporal budget constraint,

$$S_t(M^{s,H}_{F,t} + P_{BF,t}B_{F,t}) = S_t(M^{s,H}_{F,t-1} + B_{F,t-1}) + P_{H,t}Y_t - P_tC_t. \tag{64}$$

As discussed in Erceg, Gust and López-Salido (2009), in the model of complete financial market, where net foreign assets do not accumulate, it is not necessary to characterize the current account dynamics in order to characterize the equilibrium allocations. Namely, the standard IS curve provides a complete description of aggregate demand. However, in the model of incomplete financial market, the standard IS curve only determines how aggregate demand grows over time, while the current level is pinned down by the accumulation of net foreign assets.

2.8 Log-liner approximations

We log-linear approximate our model around its initial symmetric steady state, which is derived in Appendix 1. We denote an arbitrary variable X_t as $X_t = X(1+x_t)$, where lower case letter x_t denotes the percentage deviation from its steady state value. As for interest rates, we define as $\hat{i}_{H,t} = i_{H,t} - i_H$, $\hat{i}^*_{F,t} = i^*_{F,t} - i^*_F$ and $\hat{r}^n_t = r^n_t - r^n$.

At first, intratemporal optimal condition (13) and Euler equation (14) are log-linearized respectively as

$$\varphi n^s_t - u_{c,t} = w_t - p_t \tag{65}$$

$$u_{c,t} = E_t[u_{c,t+1}] + (i_{H,t} - E_t[\pi_{t+1}]). \tag{66}$$

As shown in Appendix 2, the marginal utility of consumption $u_{c,t}$ is calculated as,

$$u_{c,t} = -\sigma c_t - d_5\left\{(1-\delta)\hat{i}_{H,t} + \delta\hat{i}_{F,t}\right\} \tag{67}$$

where

$$d_5 = \left(\frac{1}{\theta} - \sigma\right)(1-d_3)\frac{\theta\beta}{1-\beta}$$

and

$$\delta = \frac{SM_F / P}{M_H / P + SM_F / P}.$$

Equation (67) is a key equation of currency substitution. We can see that the higher the degree of currency substitution δ, the less effect the home nominal interest rate $\hat{i}_{H,t}$ has on the marginal utility of consumption $u_{c,t}$, while the larger effect the foreign nominal interest rate $\hat{i}_{F,t}$ has. Under the perfect currency substitution $\delta = 1$, the home interest rate has no effects on $u_{c,t}$. Suppose that $\hat{i}_{H,t}$ rises, then currency index z_t falls (equation (A29)). Here, the effect of the fall in z_t on $u_{c,t}$ depends on the relative size of intertemporal elasticity of substitution σ and the inverse of intratemporal elasticity of substitution $1/\theta$. If $1/\theta > \sigma$ ($d_5 > 0$), namely, consumption and currency index are complements, the fall in z_t leads to a reduction in $u_{c,t}$ (equations (A30)), which induces an increase in real wages $w_t - p_t$ (equation (65)). Then, it raises firm's marginal cost and therefore inflation rate, and has negative impacts on output and

consumption. An opposite pattern emerges when $1/\theta < \sigma$ ($d_5 < 0$), namely, they are substitutes.

Plugging equation (67) into Euler equation (66), we obtain

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(i_{H,t} - E_t[\pi_{t+1}]) + \frac{d_5}{\sigma}\left\{(1-\delta)\Delta\hat{i}_{H,t+1} + \delta\Delta\hat{i}_{F,t+1}\right\}. \tag{68}$$

Log-linear approximations of the terms of trade (32) and the law of one prices (34) are

$$\tau_t = p_{H,t} - p_{F,t} \tag{69}$$

$$p_{H,t} = s_t + p^*_{H.t},\ \ p_{F,t} = s_t + p^*_{F,t}. \tag{70}$$

Using equations (69) and (70), the home consumer price index (11) can be represented as

$$\begin{aligned} p_t &= (1-a)p_{H,t} + ap_{F,t},\ \ as\ \ n \to 0 \\ &= (1-a)p_{H,t} + a(s_t + p^*_{F,t}) \\ &= p_{H,t} - a\tau_t \\ &= (1-a)\tau_t + p_{F,t} \end{aligned} \tag{71}$$

Similarly, the foreign consumer price index (27) is

$$p^*_t = p^*_{F,t},\ \ as\ \ n \to 0. \tag{72}$$

which states that due to the small open country assumption, the foreign consumer price index is approximated by aggregate price index of goods *F*. Taking the first difference of equation (71) and using equation (72), an overall inflation rate in home country is

$$\begin{aligned} \pi_t &= (1-a)\pi_{H,t} + a\pi_{F,t} \\ &= (1-a)\pi_{H,t} + a(\Delta s_t + \pi^*_t) \\ &= \pi_{H,t} - a\Delta\tau_t \\ &= (1-a)\Delta\tau_t + \pi_{F,t}. \end{aligned} \tag{73}$$

Using equation (71), the real exchange rate (35) can be written as

$$\begin{aligned} q_t &= s_t + p^*_t - p_t \\ &= -(1-a)\tau_t \\ &= -\frac{1-a}{a}(p_{H,t} - p_t). \end{aligned} \tag{74}$$

Log-linear approximations of aggregate production function (55) and (37) are

$$y_t = a_t + n_t \tag{75}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_{A,t}. \tag{76}$$

Log-linearizing labor market clearing condition (61), and combining it with equations (67), (74) and (75), the real marginal cost can be expressed as

$$\begin{aligned} \phi_t &= \varphi n_t - u_{c,t} + p_t - p_{H,t} - a_t \\ &= \varphi y_t + \sigma c_t + d_5\left\{(1-\delta)\hat{i}_{H,t} + \delta\hat{i}_{F,t}\right\} + \frac{a}{1-a}q_t - (1+\varphi)a_t. \end{aligned} \tag{77}$$

The price index of goods *H* (40) and the firm's profit maximization condition (44) are,

$$\hat{p}^+_{H,t} = \frac{\chi}{1-\chi}\pi_{H,t} \tag{78}$$

$$\hat{p}^+_{H,t} = (1-\beta\chi)\phi_t + \beta\chi E_t\left[p^+_{H,t+1}\right] + \beta\chi E_t[\pi_{H,t+1}] \tag{79}$$

where $\hat{p}^+_{H,t}$ denotes percent deviation of $P^+_{H,t}/P_{H,t}$ from its initial steady state value. Combining two equations above, the marginal cost-based New Keynesian Phillips Curve can be obtained as

$$\pi_{H,t} = \lambda\phi_t + \beta E_t[\pi_{H,t+1}] \tag{80}$$

where

$$\lambda = \frac{(1-\chi)(1-\beta\chi)}{\chi}.$$

Using equation (74), the market clearing condition of goods H is

$$\begin{aligned} y_t &= -\eta(p_{H,t} - p_t) + (1-a)c_t + ac_t^* + a\eta q_t \\ &= (1-a)c_t + ac_t^* + \frac{(2-a)a\eta}{1-a} q_t \end{aligned} \tag{81}$$

which means that home output is a weighted average of home and foreign expenditures, plus an expenditure switching factor, which is proportional to the real exchange rate.

Monetary policy rule is

$$\hat{i}_{H,t} = \hat{r}_t^n + \psi_x x_t + \psi_\pi \pi_t + \psi_q (q_t - q_t^n) + \nu_{iH,t} \tag{82}$$

where $\nu_{iH,t}$ is governed by equation (52). $x_t = y_t - y_t^n$ denotes the output gap, which is defined by the difference between the actual and natural level output. Similarly, $q_t - q_t^n$ corresponds to the real exchange rate gap.

As for the foreign economy, the Euler equation (31) is

$$u_{c^*,t} = E_t[u_{c^*,t+1}] + (i_{F,t}^* - E_t[\pi_{t+1}^*]) \tag{83}$$

where the marginal utility of consumption is

$$u_{c^*,t} = -\sigma c_t^*$$

Using equations (67) and (83), log-linear approximation of risk-sharing condition (49) is

$$\begin{aligned} E_t[u_{c,t+1} - u_{c,t}] &= E_t[u_{c^*,t+1} - u_{c^*,t}] - E_t[\Delta q_{t+1}] + \kappa nfa_t \\ q_t &= E_t[q_{t+1}] - \sigma(E_t[\Delta c_{t+1}] - E_t[\Delta c_{t+1}^*]) - d_5\left\{(1-\delta)\Delta\hat{i}_{H,t+1} + \delta\Delta\hat{i}_{F,t+1}\right\} - \kappa nfa_t. \end{aligned} \tag{84}$$

where $nfa_t \equiv S_t(B_{F,t} + M_{F,t})/P_H Y$ denotes net foreign assets. We ignore an uninteresting term relating to M_F. Modified uncovered interest parity condition (50) is

$$\hat{i}_{H,t} = \hat{i}_{F,t}^* + E_t[\Delta s_{t+1}] - \kappa nfa_t \tag{85}$$

where risk premium is given by

$$\hat{i}_{F,t} = \hat{i}_{F,t}^* - \kappa nfa_t.$$

Lastly, as shown in the Appendix 3, the dynamics of net foreign assets is given by

$$\beta nfa_t = nfa_{t-1} + p_{H,t} + y_t - p_t - c_t \tag{86}$$

2.9 Tractable representations

Here, we will follow Malik (2005) and Felices and Tuesta (2007) to derive tractable representations of our model. Specially mentioned, our model includes the real exchange rate gap as an endogenous variable.

At first, we will derive New Keynesian Phillips Curve. Inserting the market clearing condition of goods H (81) into equation (77), the real marginal cost can be rewritten as

$$\phi_t = \left(\varphi + \frac{\sigma}{1-a}\right) y_t - \frac{\sigma a}{1-a} c_t^* - \frac{a+d_6}{(1-a)^2} q_t + d_5\left\{(1-\delta)\hat{i}_{H,t} + \delta\hat{i}_{F,t}\right\} - (1+\varphi)a_t \tag{87}$$

where

$$d_6 = a(2-a)(\eta\sigma - 1).$$

For Taylor rule, we require the output gap and the real exchange rate gap which are defined by the differences between economy under the sticky price and natural level output under the frictionless economy, where financial market is complete, price is flexible price and money is neutral. The natural output y_t^n and the natural real exchange rate q_t^n can be obtained by setting $\phi_t = 0$ and $\hat{i}_{H,t} = \hat{i}_{F,t} = 0$ for all t [5],

$$0=\left(\varphi+\frac{\sigma}{1-a}\right)y_t^n-\frac{a\sigma}{1-a}c_t^*-\frac{a+d_6}{(1-a)^2}q_t^n-(1+\varphi)a_t\,. \tag{88}$$

Then, subtracting equation (88) from (87), equation (87) can be expressed in terms of output gap and the real exchange rate gap as

$$\phi_t=\left(\varphi+\frac{\sigma}{1-a}\right)x_t-\frac{a+d_6}{(1-a)^2}(q_t-q_t^n)+d_5\left\{(1-\delta)\hat{i}_{H,t}+\delta\hat{i}_{F,t}\right\}. \tag{89}$$

Plugging equation (89) into (80), we can obtain New Keynesian Phillips Curve,

$$\pi_{H,t}=\lambda\left(\varphi+\frac{\sigma}{1-a}\right)x_t-\frac{\lambda(a+d_6)}{(1-a)^2}(q_t-q_t^n)+\lambda d_5\left\{(1-\delta)\hat{i}_{H,t}+\delta\hat{i}_{F,t}\right\}+\beta E_t[\pi_{H,t+1}]. \tag{90}$$

Equation (90) is also a key equation of currency substitution. It means that as the degree of currency substitution δ becomes higher, the ability of the central bank to stabilize inflation rate using home interest rate diminishes.

Next, we will derive IS equation. Using the definition of the real exchange rate, modified uncovered interest parity condition (85) can be written in terms of the real interest rate parity condition,

$$\begin{aligned}E_t[\Delta q_{t+1}]&=(i_{H,t}-E_t[\pi_{t+1}])-(i_{F,t}-E_t[\pi_{t+1}^*])+\kappa nfa_t\\&=\left\{i_{H,t}-\left(E_t[\pi_{H,t+1}]+\frac{a}{1-a}E_t[\Delta q_{t+1}]\right)\right\}-(i_{F,t}-E_t[\pi_{t+1}^*])+\kappa nfa_t.\end{aligned}$$

Therefore,

$$E_t[\Delta q_{t+1}]=(1-a)\left\{(i_{H,t}-E_t[\pi_{H,t+1}])-(i_{F,t}-E_t[\pi_{t+1}^*])+\kappa nfa_t\right\}. \tag{91}$$

Plugging market clearing condition of goods H (81) and (91) into Euler equation (68), we obtain

$$\begin{aligned}y_t&=E_t[y_{t+1}]-d_6E_t[\Delta c_{t+1}^*]-\frac{(1+d_6)}{\sigma}(i_{H,t}-E_t[\pi_{t+1}])\\&\quad+\frac{d_5(1-a)}{\sigma}\left\{(1-\delta)\Delta\hat{i}_{H,t+1}+\delta\Delta\hat{i}_{F,t+1}\right\}-\frac{(a+d_6)\kappa}{\sigma}nfa_t.\end{aligned}$$

In terms of output gap, above equation can be rewritten as

$$\begin{aligned}x_t&=E_t[x_{t+1}]-\frac{(1+d_6)}{\sigma}(i_{H,t}-E_t[\pi_{t+1}]-\hat{r}_t^n)\\&\quad+\frac{d_5(1-a)}{\sigma}\left\{(1-\delta)\Delta\hat{i}_{H,t+1}+\delta\Delta\hat{i}_{F,t+1}\right\}-\frac{(a+d_6)\kappa}{\sigma}nfa_t\end{aligned} \tag{92}$$

where natural real interest rate $\hat{r}_t^n$ is defined by

$$\hat{r}_t^n=\frac{\sigma}{1+d_6}\left\{E_t[\Delta y_{t+1}^n]+d_6E_t[\Delta c_{t+1}^*]\right\}. \tag{93}$$

Note that the IS curve (92) includes a net foreign assets term.

Next, we will derive a dynamic behavior of the real exchange rate gap. Combining the market clearing condition of goods H (81) and risk sharing condition (84) yields

$$\begin{aligned}q_t&=E_t[q_{t+1}]-\frac{\sigma(1-a)}{1+d_6}(E_t[\Delta y_{t+1}]-E_t[\Delta c_{t+1}^*])\\&\quad-\frac{(1-a)^2d_5}{1+d_6}\left\{(1-\delta)E_t[\Delta\hat{i}_{H,t+1}]+\delta E_t[\Delta\hat{i}_{F,t+1}]\right\}-\frac{\kappa(1-a)^2}{1+d_6}nfa_t.\end{aligned}$$

Expressing above equation in terms of output gap and real exchange rate gap, we obtain

$$q_t - q_t^n = E_t[q_{t+1} - q_t^n] - \frac{\sigma(1-a)}{1+d_6} E_t[\Delta x_{t+1}] \\ - \frac{(1-a)^2 d_5}{1+d_6}\left\{(1-\delta)E_t[\Delta \hat{i}_{H,t+1}] + \delta E_t[\Delta \hat{i}_{F,t+1}]\right\} - \frac{\kappa(1-a)^2}{1+d_6} nfa_t \tag{94}$$

where the natural real exchange rate q_t^n is defined by

$$E_t[\Delta q_{t+1}^n] = \frac{\sigma(1-a)}{1+d_6}\left(E_t[\Delta y_{t+1}^n] - E_t[\Delta c_{t+1}^*]\right) \\ = (1-a)\left\{\hat{r}_t^n - (i_{F,t}^* - E_t[\pi_{t+1}^*])\right\}. \tag{95}$$

Equation (95) can be interpreted as equation (91) evaluated at the frictionless economy in which price is flexible and financial market is complete. Equation (94) states that the relative size of the inverse of elasticity of substitution between consumption and currency index $1/\theta$ and the coefficient of risk aversion σ (the sign of d_5) determines the dynamic behavior of the real exchange rate gap, $q_t - q_t^n$. If $1/\theta > \sigma$ ($d_5 > 0$, complements), a rise in $\hat{i}_{H,t}$ decreases $u_{c,t}$, which leads to a depreciation of $q_t - q_t^n$ through the risk sharing condition (equation (84)). An opposite pattern emerges when $1/\theta < \sigma$ ($d_5 < 0$, substitutes).

Here, we will derive the natural real interest rate. Taking the first difference of equation (88) and using equation (95), we can obtain

$$E_t[\Delta y_{t+1}^n] = \frac{(1+\varphi)(1+d_6)}{\varphi(1+d_6)+\sigma}[\Delta a_{t+1}] - \frac{\sigma d_6}{\varphi(1+d_6)+\sigma} E_t[\Delta c_{t+1}^*].$$

Thus, natural real interest rate can be derived as

$$\hat{r}_t^n = \frac{\sigma}{\varphi(1+d_6)+\sigma}\left\{(1+\varphi)[\Delta a_{t+1}] + \varphi d_6 E_t[\Delta c_{t+1}^*]\right\} \\ = \frac{1}{\varphi(1+d_6)+\sigma}\left\{-\sigma(1+\varphi)(1-\rho_A)a_t + \varphi d_6(\hat{i}_{F,t}^* - E_t[\pi_{t+1}^*])\right\} \tag{96}$$

where the second equality follows from foreign Euler equations (83).

As for the current account, by making use of equations (74) and market clearing condition of goods *H* (81), equation (86) can be rewritten as,

$$\beta nfa_t = nfa_t - \frac{a}{1-a}(y_t - c_t^*) + \frac{a}{1-a}\left\{\frac{(2-a)\eta}{1-a} - 1\right\} q_t. \tag{97}$$

Evaluating equation (97) at the frictionless economy leads to[6]

$$0 = -\frac{a}{1-a}(y_t^n - c_t^*) + \frac{a}{1-a}\left\{\frac{(2-a)\eta}{1-a} - 1\right\} q_t^n. \tag{98}$$

Then, subtracting equation (98) from (97), equation (97) can be expressed in terms of output gap and the real exchange rate gap as

$$\beta nfa_t = nfa_{t-1} - \frac{a}{1-a} x_t + \frac{a}{1-a}\left\{\frac{(2-a)\eta}{1-a} - 1\right\}(q_t - q_t^n). \tag{99}$$

Finally, following Felices and Tuesta (2007), we assume that the foreign nominal interest rate follows an exogenous process given by

$$\hat{i}_{F,t}^* = \rho_{i_F^*} \hat{i}_{F,t-1}^* + \varepsilon_{i_F^*,t} \tag{100}$$

where $0 \le \rho_{i_F^*} \le 1$ is an autoregressive coefficient and $\varepsilon_{i_F^*,t}$ denotes random variable serially uncorrelated and normally distributed, with zero mean and constant variance $\sigma_{i_F^*}^2$. For simplicity, we also assume that $\pi_t^* = 0$.

2.10 Recapitulation

Our model is constituted by following twelve equations with twelve endogenous variables $\{x_t, \pi_t, \pi_{H,t}, s_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{r}_t^n, \nu_{iH,t}, a_t, q_t - q_t^n, nfa_t, i_{F,t}^*\}$ and three exogenous shocks $\{\varepsilon_{A,t}, \varepsilon_{iH,t}, \varepsilon_{iF^*,t}\}$.

IS curve

$$x_t = E_t[x_{t+1}] - \frac{(1+d_6)}{\sigma}(i_{H,t} - E_t[\pi_{t+1}] - r_t^n) + \frac{d_5(1-a)}{\sigma}\left\{(1-\delta)\Delta\hat{i}_{H,t+1} + \delta\Delta\hat{i}_{F,t+1}\right\} - \frac{(a+d_6)\kappa}{\sigma} nfa_t \tag{92}$$

Natural real interest rate

$$\hat{r}_t^n = \frac{1}{\varphi(1+d_6)+\sigma}\left\{-\sigma(1+\varphi)(1-\rho_A)a_t + \varphi d_6 \hat{i}_{F,t}^*\right\} \tag{96}$$

Home productivity

$$a_t = \rho_A a_{t-1} + \varepsilon_{A,t} \tag{76}$$

New Keynesian Phillips Curve

$$\pi_{H,t} = \lambda\left(\varphi + \frac{\sigma}{1-a}\right)x_t - \frac{\lambda(a+d_6)}{(1-a)^2}(q_t - q_t^n) + \lambda d_5\left\{(1-\delta)\hat{i}_{H,t} + \delta\hat{i}_{F,t}\right\} + \beta E_t[\pi_{H,t+1}] \tag{90}$$

Overall inflation rate

$$\pi_t = (1-a)\pi_{H,t} + a\Delta s_t \tag{73}$$

Home monetary policy rule

$$\hat{i}_{H,t} = \hat{r}_t^n + \psi_x x_t + \psi_\pi \pi_{H,t} + \psi_q(q_t - q_t^n) + \nu_{iH,t} \tag{82}$$

$$\nu_{iH,t} = \rho_{iH}\nu_{iH,t-1} + \varepsilon_{iH,t} \tag{52}$$

Real exchange rate gap

$$q_t - q_t^n = E_t[q_{t+1} - q_t^n] - \frac{\sigma(1-a)}{1+d_6}E_t[\Delta x_{t+1}] - \frac{(1-a)^2 d_5}{1+d_6}\left\{(1-\delta)E_t[\Delta\hat{i}_{H,t+1}] + \delta E_t[\Delta\hat{i}_{F,t+1}]\right\} - \frac{\kappa(1-a)^2}{1+d_6} nfa_t \tag{94}$$

Modified uncovered interest parity

$$\hat{i}_{H,t} = \hat{i}_{F,t}^* + E_t[\Delta s_{t+1}] - \kappa nfa_t \tag{85}$$

Risk premium

$$\hat{i}_{F,t} = \hat{i}_{F,t}^* - \kappa nfa_t$$

Net foreign asset

$$\beta nfa_t = nfa_{t-1} - \frac{a}{1-a}x_t + \frac{a}{1-a}\left\{\frac{(2-a)\eta}{1-a} - 1\right\}(q_t - q_t^n) \tag{99}$$

Foreign monetary policy rule

$$\hat{i}_{F,t}^* = \rho_{i_F^*}\hat{i}_{F,t-1}^* + \varepsilon_{i_F^*,t} \tag{100}$$

3. Calibration

In this section, we present quantitative results based on calibration analyses. We will investigate the extent to which the degree of currency substitution influences on the effects of monetary policy. We also calculate unconditional variances for key variables under the different degree of currency substitution.

3.1 Directions of foreign monetary policy shock

At first, we summarize the effects of monetary policies on x_t, $\pi_{H,t}$ and $q_t - q_t^n$. There are mainly two direct channels through which monetary policies are transmitted to the home economy.

The first one is a real interest rate channel. As for the home monetary policy, a rise in $\hat{i}_{H,t}$ directly increases real interest rate, which has a negative pressure on x_t (equation (92)) and an appreciation pressure on $q_t - q_t^n$. To see the latter, combining equations (91) and (95) yields

$$E_t[\Delta q_{t+1} - \Delta q_{t+1}^n] = (1-a)\left\{(i_{H,t} - E_t[\pi_{H,t+1}]) - \hat{r}_t^n\right\}.$$

From above equation, we can confirm that a rise in $\hat{i}_{H,t}$ leads to an appreciation of $q_t - q_t^n$. As for the foreign monetary policy, a rise in $\hat{i}_{F,t}^*$ increases $\hat{r}_t^n$ (equation (96)), which has a positive pressure on x_t (equation (92)) and an depreciation pressure on $q_t - q_t^n$.

The second one is a marginal utility of consumption channel, which is captured by terms, $(1-\delta)\hat{i}_{H,t+1} + \delta\hat{i}_{F,t+1}$ or $(1-\delta)\Delta\hat{i}_{H,t+1} + \delta\Delta\hat{i}_{F,t+1}$. Its direction depends on some key parameters; including the relative risk aversion σ, the elasticity of substitution between consumption and currency index θ, the weight on currency H in the currency index γ and the elasticity of currency substitution between currency H and F ν. As discussed in Section 2, the relative size of $1/\theta$ and σ determines whether consumption and the currency index are complements or substitutes. As for the home monetary policy, if $1/\theta > \sigma$ ($d_5 > 0$,complements), a rise in $\hat{i}_{H,t}$ decreases $u_{c,t}$, which leads to a fall in x_t, a rise in $\pi_{H,t}$ and a depreciation of $q_t - q_t^n$. If $1/\theta < \sigma$ ($d_5 < 0$, substitutes), opposite patterns could emerge. Moreover, the degree of currency substitution in the steady state δ depends on parameters γ and ν, which determines the magnitude of this channel. The lower value of γ and/or the higher value of ν means higher value of δ, which diminishes the effects of home monetary policy ($\hat{i}_{H,t}$). Similar arguments can be applied for foreign monetary policy.

3.2 Impulse responses

To highlight the extent to which the degree of currency substitution influences on the effects of monetary policies, we perform the calibration analyses. For parameter values, standard baseline values appeared in Malik (2005) and Felices and Tuesta (2007) are chosen.

We set a discount factor $\beta = 0.99$, which implies an annualized rate of interest rate of 4 percent in the steady state. The share of foreign goods in consumption is set to $a = 0.4$, which is said to be typical in small open economy. We set the coefficient risk averse $\sigma = 1$ (which corresponds to log utility) and the inverse of elasticity of labor supply $\varphi = 1$ to isolate the role of foreign currency in the currency index and non-separability of the currency index from consumption. The weight on consumption in the consumption-currency index X_t is set equal to $\omega = 0.8$. The elasticity of substitution between goods H and F η is set to equal to 3.0. The elasticity of currency substitution between currency H and F is set to equal to $\nu = 3.0$. The probability with which firms cannot change their prices is set equal to $\chi = 0.75$, which is consistent with an average period of one year between price adjustments. We set a parameter of the risk premium to $\kappa = 0.01$, which implies that if the net foreign assets per nominal GDP would increase by 1 percent point, then the risk premium of $\hat{i}_{F,t}$ over $\hat{i}_{F,t}^*$ would decrease by 1 basis-point. For the monetary policy rule, we set $\psi_x = 0.5$, $\psi_\pi = 1.8$ and $\psi_q = 0.5$ with persistent

parameter $\rho_{iH} = 0.7$. Persistent parameters of productivity and foreign monetary policy are $\rho_A = 0.7$ and $\rho_{i_F^*} = 0.7$. As for the variances of exogenous shocks, productivity shock σ_A^2, home monetary policy shock σ_{iH}^2 and foreign monetary policy shock $\sigma_{i_F^*}^2$ are all taken to be $(0.009)^2$.

As mentioned above, the key parameters of our model are the relative risk aversion σ, the elasticity of substitution between consumption and currency index θ, the weight on currency H in the currency index γ and the elasticity of currency substitution between currency H and F ν. Therefore, we calibrate our model under six possible scenarios. Namely, we assume two cases in which (1) $1/\theta > \sigma$ (complement) and (2) $1/\theta < \sigma$ (substitute). For each case, we also consider three different degrees of currency substitution, where (H) $\gamma = 0.4$, then $\delta = 0.77$ (high degree of currency substitution), (M) $\gamma = 0.5$, then $\delta = 0.5$ (middle degree of currency substitution) and (L) $\gamma = 0.6$, then $\delta = 0.23$ (low degree of currency substitution). These parameters are summarized in Table 1.

Table1 Calibration parameters

β	discount factor	0.99
σ	coefficient of risk aversion	1.0
φ	inverse of elasticity of labor supply	1.0
θ	elasticity of substitution between consumption and currency index	0.8, 1.2
ω	weight on consumption in the consumption-currency index	0.8
ν	elasticity of currency substitution between currency H and F	3.0
γ	weight on currency H in the currency index	0.4, 0,5 ,0.6
a	share of foreign goods in consumption	0.4
η	elasticity of substitution between goods H and F	3.0
χ	probability with which firms cannot change their prices	0.75
κ	coefficient of risk premium	0.01
ψ_x	coefficient on output gap in the home monetary policy rule	0.5
ψ_π	coefficient on inflation rate in the home monetary policy rule	1.8
ψ_q	coefficient on real exchange rate in the home monetary policy rule	0.5
ρ_{iH}	nominal interest rate smoothing parameter in the home monetary policy rule	0.7
ρ_A	persistent parameter of productivity shock	0.7
$\rho_{i_F^*}$	persistent parameter of foreign monetary policy shock	0.7
σ_{iH}^2	variance of home monetary policy shock	$(0.009)^2$
σ_A^2	variance of productivity shock	$(0.009)^2$
$\sigma_{i_F^*}^2$	variance of foreign monetary policy shock	$(0.009)^2$

Figure 1 displays impulse responses of x_t, $\pi_{H,t}$, $q_t - q_t^n$, $\hat{i}_{H,t}$ and nfa_t with respect to home monetary policy shock. Figure1-1 is for the case of $1/\theta > \sigma$ (complements), and Figure 1-2 is for the case of $1/\theta < \sigma$ (substitutes). From Figure 1, we can see that whether consumption and currency index are complement or substitutes does not influence the dynamic behaviors of x_t, $\pi_{H,t}$ and $q_t - q_t^n$. In all cases, both x_t and $\pi_{H,t}$ decrease and $q_t - q_t^n$ appreciates following the contractionary monetary policy shock. These results show that the real interest rate channel dominates the marginal utility of consumption channel. Another important point to note is that $\hat{i}_{H,t}$ decreases in the case of $1/\theta > \sigma$ (complements), while it increases in the case of $1/\theta < \sigma$ (substitutes), following the contractionary home monetary policy shock. We can also see that the degree of currency substitution does not have serious impacts on the effects of home monetary policy. Impulse responses are not so different regardless of the different degrees of currency

substitution. This is because while the degree of currency substitution (δ) can influence on the effects of monetary policy through the marginal utility of consumption channel, but it is dominated by the real interest rate channel. Therefore, we can say that the degree of currency substitution does not influence on the effects of home monetary policy. We can also say that the real exchange rate has a tendency to appreciate following the contractionary home monetary policy shock under the plausible parameter set.

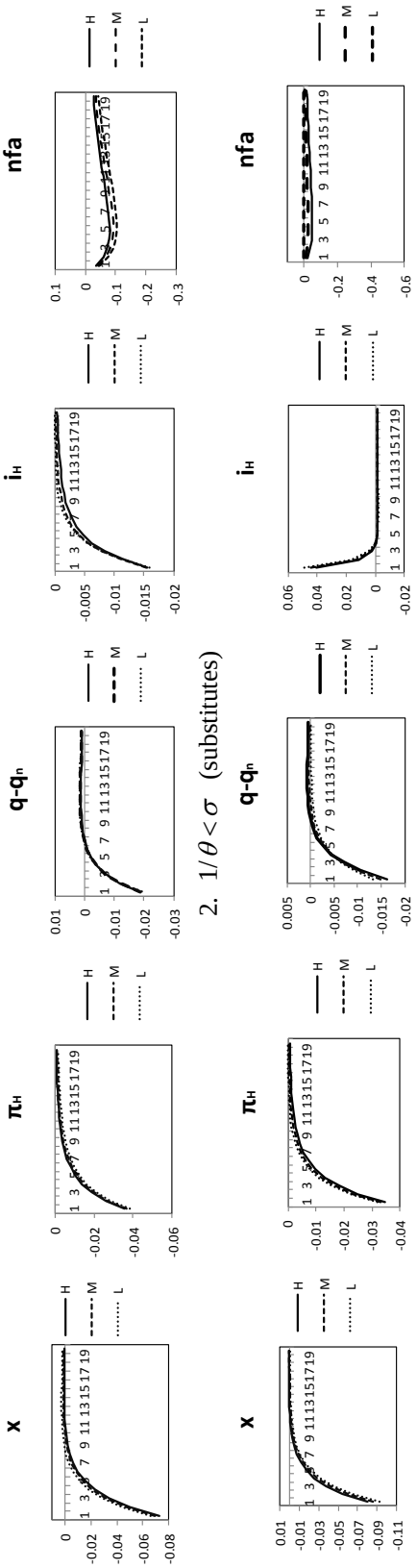

Figure 1 Impulse responses to *home* monetary policy shock

1. $1/\theta > \sigma$ (complement)

2. $1/\theta < \sigma$ (substitutes)

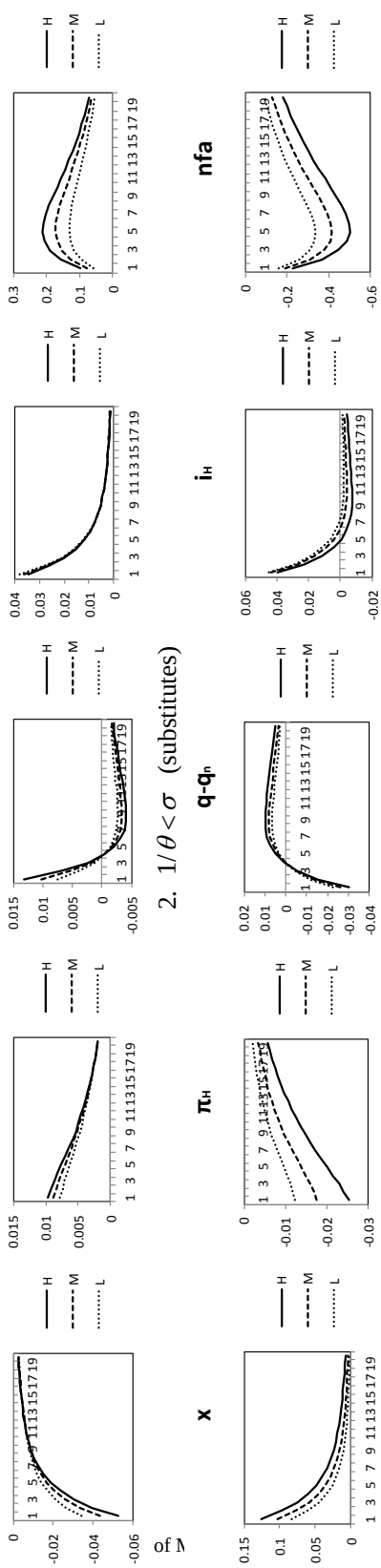

Figure 2 Impulse responses to *foreign* monetary policy shock

1. $1/\theta > \sigma$ (complement)

2. $1/\theta < \sigma$ (substitutes)

On the other hand, Figures 2, showing impulse responses with respect to foreign monetary policy shock, highlights that whether consumption and currency index are complement or substitutes does influence on the direction of monetary policy. In the case of $1/\theta > \sigma$ (complement), x_t decreases, $\pi_{H,t}$ increases and $q_t - q_t^n$ depreciates. On the other hand, in the case of $1/\theta < \sigma$ (substitutes), the impulse responses show opposite patterns. These results are consistent with those explained by the marginal utility of consumption channel. We can also see that the degree of currency substitution does have meaningful impacts on the foreign monetary policy effects. The larger the degree of currency substitution δ, the larger the impact of the foreign monetary policy shock.

Moreover, in the complete financial market model presented by Felices and Tuesta (2007), x_t, $\pi_{H,t}$ and $q_t - q_t^n$ revert to their initial steady states (namely, zero) almost after six periods following foreign monetary policy shock. On the other hand, in our incomplete financial market model, x_t reverts to the level below zero (negative output gap) and $q_t - q_t^n$ also reverts to the level below zero (appreciation) in the case of $1/\theta > \sigma$ (complement). As for $\pi_{H,t}$, it increases after the foreign monetary policy shock and remains at the higher level in the long-run. These results are due to the existence of net foreign assets. As discussed above, in the case of $1/\theta > \sigma$ (complement), a rise in $\hat{i}_{F,t}^*$ leads to a fall in x_t and a depreciation of $q_t - q_t^n$ in the short-run, which brings an increase in nfa_t. To maintain nfa_t positive, x_t must become negative and $q_t - q_t^n$ must appreciate in the new steady state. The upward pressure on $\pi_{H,t}$ from the appreciation of $q_t - q_t^n$ would mitigate the downward pressure on $\pi_{H,t}$ from the fall in x_t, thus $\pi_{H,t}$ remains at the higher level in the new steady state. It means that different degree of currency substitution has different permanent effects.

3.3 Unconditional Variances

Table 2 reports unconditional variances of x_t, $\pi_{H,t}$, $q_t - q_t^n$, $\hat{i}_{H,t}$ and nfa_t with respect to home monetary policy shock. Table 2-1 is for the case of $1/\theta > \sigma$ (complements), and Table 2-2 is for the case of $1/\theta < \sigma$ (substitutes). From Table 2, we can see that unconditional variances following the home monetary policy shock are relatively stable regardless of the different degrees of currency substitution. It implies that the degree of currency substitution does not have serious impacts on the home monetary policy effects. An extra point to note is that in the case of $1/\theta > \sigma$ (complements), unconditional variance of x_t is highest in the case of (H), while that of $\pi_{H,t}$ is highest in the case of (L). The opposite patterns can be seen in the case of $1/\theta < \sigma$ (substitutes). The reasons are as follows. In the case of $1/\theta > \sigma$ (complements), both the real interest channel and the marginal utility of consumption channel have a negative pressure on x_t, which would magnify the unconditional variance of x_t. On the other hand, the real interest channel has a negative pressure, while the marginal utility of consumption channel has a positive pressure on $\pi_{H,t}$. Each pressure would offset the other, which would diminish the unconditional variance of $\pi_{H,t}$. The opposite explanations can be possible in the case of $1/\theta < \sigma$ (substitutes)

On the other hand, Table 3, showing unconditional variances with respect to foreign monetary policy shock, highlights that that foreign monetary policy shock is sensitive to the degree of currency substitution. For example, as δ increases from $\delta = 0.23$ to $\delta = 0.77$, unconditional variances of x_t, $\pi_{H,t}$, $q_t - q_t^n$ increase significantly in all cases. The implied increase of δ can be obtained by small increase of γ from 0.4 to 0.6. These findings point out that the small rise in the degree of currency substitution would destabilize the economy.

Table 2. Unconditional variances to *home* monetary policy shock

2-1. The case for $1/\theta > \sigma$ (complement)

ψ_q	Degree of currency substituion	var(x)	var(π_H)	var(q-q_n)	var(i_H)	var(nfa)
	High	0.0123	0.0031	0.0007	0.0005	0.0843
$\psi_q = 0$	Middle	0.0113	0.0033	0.0008	0.0005	0.1114
	Low	0.0103	0.0035	0.0008	0.0006	0.1451
	High	0.0098	0.0028	0.0006	0.0005	0.0710
$\psi_q = 0.5$	Middle	0.0088	0.0031	0.0006	0.0006	0.0977
	Low	0.0077	0.0035	0.0006	0.0008	0.1348
	High	0.0080	0.0026	0.0005	0.0006	0.0609
$\psi_q = 1$	Middle	0.0070	0.0029	0.0005	0.0007	0.0872
	Low	0.0058	0.0036	0.0005	0.0010	0.1279
	High	0.0067	0.0024	0.0004	0.0006	0.0530
$\psi_q = 1.5$	Middle	0.0056	0.0029	0.0004	0.0008	0.0789
	Low	0.0045	0.0039	0.0004	0.0014	0.1242

2-2.The case for $1/\theta < \sigma$ (substitutes)

ψ_q	Degree of currency substituion	var(x)	var(π_H)	var(q-q_n)	var(i_H)	var(nfa)
	High	0.0142	0.0029	0.0006	0.0006	0.0394
$\psi_q = 0$	Middle	0.0167	0.0026	0.0005	0.0005	0.0101
	Low	0.0196	0.0023	0.0004	0.0006	0.0000
	High	0.0119	0.0026	0.0005	0.0005	0.0310
$\psi_q = 0.5$	Middle	0.0143	0.0022	0.0004	0.0005	0.0077
	Low	0.0169	0.0020	0.0003	0.0005	0.0000
	High	0.0102	0.0023	0.0004	0.0005	0.0250
$\psi_q = 1$	Middle	0.0124	0.0020	0.0003	0.0004	0.0060
	Low	0.0148	0.0018	0.0003	0.0004	0.0000
	High	0.0088	0.0020	0.0003	0.0005	0.0205
$\psi_q = 1.5$	Middle	0.0109	0.0017	0.0003	0.0004	0.0048
	Low	0.0130	0.0015	0.0003	0.0004	0.0000

Table 3. Unconditional variances to *foreign* monetary policy shock

3-1. the case for $1/\theta > \sigma$ (complement)

ψ_q	Degree of currency substituion	var(x)	var(π_H)	var(q-q_n)	var(i_H)	var(nfa)
	High	0.0056	0.0004	0.0005	0.0023	0.5028
$\psi_q = 0$	Middle	0.0040	0.0003	0.0003	0.0025	0.3398
	Low	0.0025	0.0002	0.0002	0.0027	0.2012
	High	0.0070	0.0007	0.0004	0.0030	0.4894
$\psi_q = 0.5$	Middle	0.0052	0.0006	0.0003	0.0032	0.3445
	Low	0.0035	0.0005	0.0002	0.0034	0.2160
	High	0.0083	0.0011	0.0003	0.0037	0.4805
$\psi_q = 1$	Middle	0.0064	0.0010	0.0002	0.0040	0.3517
	Low	0.0047	0.0010	0.0001	0.0043	0.2345
	High	0.0095	0.0015	0.0003	0.0046	0.4746
$\psi_q = 1.5$	Middle	0.0075	0.0016	0.0002	0.0050	0.3610
	Low	0.0061	0.0018	0.0001	0.0056	0.2585

3-2. The case for $1/\theta < \sigma$ (substitutes)

ψ_q	Degree of currency substituion	var(x)	var(π_H)	var(q-q_n)	var(i_H)	var(nfa)
	High	0.0400	0.0038	0.0029	0.0031	3.1772
$\psi_q = 0$	Middle	0.0243	0.0019	0.0020	0.0036	2.0895
	Low	0.0146	0.0009	0.0014	0.0038	1.3311
	High	0.0070	0.0007	0.0004	0.0030	0.4894
$\psi_q = 0.5$	Middle	0.0052	0.0006	0.0003	0.0032	0.3445
	Low	0.0035	0.0005	0.0002	0.0034	0.2160
	High	0.0464	0.0062	0.0019	0.0030	2.6663
$\psi_q = 1$	Middle	0.0277	0.0027	0.0013	0.0029	1.6530
	Low	0.0168	0.0012	0.0009	0.0032	1.0098
	High	0.0493	0.0076	0.0016	0.0033	2.4859
$\psi_q = 1.5$	Middle	0.0294	0.0031	0.0011	0.0028	1.4981
	Low	0.0179	0.0013	0.0007	0.0030	0.8928

In addition to the benchmark case, we calibrate our model for another three types of exchange rate regime, namely, $\psi_q = 0$ (flexible exchange rate regime), 1.0 and 1.5. The results are also shown in Table 2 and 3. We can confirm that our conclusion is not changed. Namely, the degree of currency substitution does not affect unconditional variances for the home monetary policy but it affects for the foreign monetary policy. But, an important point to note is that following the foreign monetary policy shock, variances of x_t decrease while those of $q_t - q_t^n$ increase as the central bank moves to flexible exchange rate regime. This result highlights the tradeoff between stabilization of the output gap and the real exchange rate gap.

4. Conclusions

In this paper, we analyze the extent to which the degree of currency substitution influences on the effects of monetary policy in the DSGE framework. Especially, we will focus on the role of real exchange rate in the transmission of monetary policy. To attain this objective, our model assumes incomplete financial markets in which risk sharing conditions and the uncovered interest rate parity condition does not hold. The prominent feature of our model is that it includes the real exchange rate gap as an endogenous variable.

From the calibration analyses, we show that whether consumption and currency index are complement and substitutes does not influence on the direction of home monetary policy. We also show that the degree of currency substitution does not have serious impacts on the effects

of home monetary policy shock. This is because the real interest rate channel dominates the marginal utility of consumption channel. Moreover, the real exchange rate has a tendency to appreciate following the contractionary home monetary policy shock. On the other hand, we find that whether consumption and currency index are complement and substitutes does influence on the direction of foreign monetary policy. We also find that the degree of currency substitution does have some impacts on the transmission of foreign monetary policy shock to the domestic economy. The larger the degree of currency substitution, the larger the impact of the foreign monetary policy shock. Moreover, from the comparisons with the complete financial market model (Felices and Tuesta (2007)), the output gap, inflation rate and the real exchange rate gap do not revert to their initial steady states even in the long-run in the incomplete financial market. These results are due to the existence of net foreign assets. It means that different degree of currency substitution has different permanent effects.

From the analyses of unconditional variances, the degree of currency substitution does not have serious impacts on the home monetary policy effects. On the other hand, foreign monetary policy shock is sensitive to the degree of currency substitution. The higher the degree of currency substitution, the larger the unconditional variances of endogenous variables. A small rise in the degree of currency substitution increases would destabilize the economy.

There are some interesting topics for future research. They include the determinacy for an instrumental monetary policy rule, the analysis of optimal targeting policy using a micro-founded loss function and empirical analysis based on our model.

Appendix1 Symmetric steady state

Consider a deterministic initial steady state in which inflation rate is zero, all endogenous variables remain constant, net foreign assets are zero and all shocks is set to zero. Moreover, we will consider the symmetric steady state in which $P_H = P_F$, $P_H / P = P_F / P = \mathrm{T} = Q = 1$ are satisfied. We also assume that $A = 1$ for simplicity.

Euler equations (14) and (15) in the steady state are reduced to

$$\beta^{-1} = 1 + i_H = 1 + i_F. \tag{A1}$$

Money demands for currency H and F in equations (18) and (19) are

$$U_{\frac{M_H^d}{P}} = \frac{i_H}{1+i_H} U_C = (1-\beta) U_C \tag{A2}$$

$$U_{\frac{SM_F^d}{P}} = \frac{i_F}{1+i_F} U_C = (1-\beta) U_C \tag{A3}$$

where marginal utilities in the steady state are given by

$$U_C = X^{\frac{1}{\theta}-\sigma} C^{-\frac{1}{\theta}} \omega \tag{A4}$$

$$U_{\frac{M_H^d}{P}} = X^{\frac{1}{\theta}-\sigma} (1-\omega)\gamma Z^{\frac{1}{\nu}-\frac{1}{\theta}} \left(\frac{M_H^d}{P} \right)^{-\frac{1}{\nu}} \tag{A5}$$

$$U_{\frac{SM_F^d}{P}} = X^{\frac{1}{\theta}-\sigma} (1-\omega)(1-\gamma) Z^{\frac{1}{\nu}-\frac{1}{\theta}} \left(\frac{SM_F^d}{P} \right)^{-\frac{1}{\nu}} \tag{A6}$$

$$X = \left[\omega C^{\frac{\theta-1}{\theta}} + (1-\omega) Z^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{A7}$$

$$Z = \left[\gamma \left(\frac{M_H^d}{P} \right)^{\frac{\nu-1}{\nu}} + (1-\gamma) \left(\frac{SM_F^d}{P} \right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} . \tag{A8}$$

The relative money demand function (20) is

$$RM \equiv \frac{SM_F^d / P}{M_H^d / P} = \left(\frac{\gamma}{1-\gamma} \right)^{-\nu} . \tag{A9}$$

The aggregate production function (55) is

$$Y = N . \tag{A10}$$

Since $\Xi_{t,t+k} = \beta^k$ holds in the steady state, the firm's profit maximizing condition (44) is

$$\Phi = \frac{\varepsilon - 1}{\varepsilon} . \tag{A11}$$

Then, we can see that labor market clearing condition (61) is

$$\frac{N^{\varphi}}{U_C} = \frac{\varepsilon - 1}{\varepsilon} . \tag{A12}$$

Market clearing conditions of goods H and F can be respectively written as

$$Y = (1-a)C + aC^* \tag{A13}$$

$$Y^* = C^* . \tag{A14}$$

Since $B_{F,t} = 0$ and $M_{F,t} = M_F$ in the initial steady state, the transaction cost is $\Psi(0) = 1$. Thus, modified uncovered interest rate parity condition (50) can be simplified to

$$i_H = i_F^* . \tag{A15}$$

Similarly, the intertemporal budget constraint (64) can be simply written as

$$Y = C \tag{A16}$$

which means that the current account is balanced in the steady state.

Solving equation (A9) for SM_F^d / P and inserting it into (A8), then combining the obtained result with (A4), (A5) and (A7) yields the steady state real money demand for currency H,

$$\frac{M_H^d}{P} = d_1 C \tag{A17}$$

where

$$d_1 = \left[\frac{(1-\beta)\omega}{(1-\omega)\gamma \left[\left\{ \gamma + (1-\gamma) \left(\frac{\gamma}{1-\gamma} \right)^{1-\nu} \right\}^{\frac{\nu}{\nu-1}} \right]^{\frac{1}{\nu} - \frac{1}{\theta}}} \right]^{-\theta} .$$

Inserting equation (A17) into (A9), the steady state real money demand for currency F is

$$\frac{SM_F^d}{P} = \left(\frac{\gamma}{1-\gamma} \right)^{-\nu} d_1 C . \tag{A18}$$

Plugging equations (A17) and (A18) into (A8), we can see that currency index is proportional to consumption,

$$Z = d_2 C \tag{A19}$$

where

$$d_2 = \left[\gamma + (1-\gamma)\left(\frac{\gamma}{1-\gamma}\right)^{1-\nu} \right]^{\frac{\nu}{\nu-1}} \quad d_1 = \left[\frac{(1-\beta)\omega}{(1-\omega)\left\{\gamma^{\nu} + (1-\gamma)^{\nu}\right\}^{\frac{1}{\nu-1}}} \right]^{-\theta}.$$

Inserting equation (A19) into (A7) and combining the obtained result with (A4), we can obtain the steady state marginal utility of consumption as

$$U_C = \left[\omega + (1-\omega) d_2^{\frac{\theta-1}{\theta}} \right]^{\frac{1-\theta\sigma}{\theta-1}} C^{-\sigma}\omega. \tag{A20}$$

By using equations (A10), (A12), (A16) and (A20), we can obtain the steady state level of consumption (output) as

$$C = \left[\frac{\omega(\varepsilon-1)}{\varepsilon} \left\{ \omega + (1-\omega) d_2^{\frac{\theta-1}{\theta}} \right\}^{\frac{1-\theta\sigma}{\theta-1}} \right]^{\frac{1}{\varphi+\sigma}}. \tag{A21}$$

As for the foreign economy, Euler equation (31) is

$$\beta^{-1} = 1 + i_F^*. \tag{A22}$$

Appendix 2. Approximations of the marginal utility of consumption and money demand functions

At first, from the specification of utility function, $u_{c,t}$ can be written as

$$u_{c,t} = \left(\frac{1}{\theta} - \sigma\right) x_t - \frac{1}{\theta} c_t \tag{A23}$$

$$x_t = d_3 c_t + (1-d_3) z_t \tag{A24}$$

where

$$d_3 = \frac{\omega C^{\frac{\theta-1}{\theta}}}{\omega C^{\frac{\theta-1}{\theta}} + (1-\omega) Z^{\frac{\theta-1}{\theta}}} = \frac{\omega}{\omega + (1-\omega) d_2^{\frac{\theta-1}{\theta}}}$$

$$z_t = d_4 (m_{H,t} - p_t) + (1-d_4)(s_t + m_{F,t} - p_t) \tag{A25}$$

where

$$d_4 = \frac{\gamma (M_H / P)^{\frac{\nu-1}{\nu}}}{\gamma (M_H / P)^{\frac{\nu-1}{\nu}} + (1-\gamma)(SM_F / P)^{\frac{\nu-1}{\nu}}} = \frac{\gamma}{\gamma + (1-\gamma)\{\gamma/(1-\gamma)\}^{1-\nu}}.$$

d_4 represents the steady state ratio of currency H in the currency index. Here, we define the steady state degree of currency substitution as

$$\delta \equiv 1 - d_4 = \frac{SM_F / P}{M_H / P + SM_F / P} = \frac{\{\gamma/(1-\gamma)\}^{-\nu}}{1 + \{\gamma/(1-\gamma)\}^{-\nu}}. \tag{A26}$$

From equation (A26), we can see that the higher the value of γ (the larger weight on currency *F*) and the higher the value of ν (the larger elasticity of currency substitution between currency *H* and *F*), the higher the degree of currency substitution in the steady state δ.

Using above equations, we will derive a tractable representation of $u_{c,t}$. Log-linearized equations (18) and (19) are

$$m_{H,t} - p_t = \frac{\nu}{\theta} c_t + \left(1 - \frac{\nu}{\theta}\right) z_t - \frac{\nu\beta}{1-\beta} \hat{i}_{H,t} \tag{A27}$$

$$s_t + m_{F,t} - p_t = \frac{\nu}{\theta} c_t + \left(1 - \frac{\nu}{\theta}\right) z_t - \frac{\nu\beta}{1-\beta} \hat{i}_{F,t} . \tag{A28}$$

Combining equations (A25) with (A27) and (A28), we obtain

$$z_t = c_t - \frac{\theta\beta}{1-\beta} \left\{ (1-\delta)\hat{i}_{H,t} + \delta \hat{i}_{F,t} \right\} . \tag{A29}$$

Inserting equation (A29) into (A24) and then, inserting obtained result into (A23) yields

$$\begin{aligned} u_{c,t} &= \left\{ \left(\frac{1}{\theta} - \sigma \right) d_3 - \frac{1}{\theta} \right\} c_t + \left(\frac{1}{\theta} - \sigma \right) z_t \\ &= -\sigma c_t - d_5 \left\{ (1-\delta)\hat{i}_{H,t} + \delta \hat{i}_{F,t} \right\} \end{aligned} \tag{A30}$$

where

$$d_5 = \left(\frac{1}{\theta} - \sigma \right) (1 - d_3) \frac{\theta\beta}{1-\beta} .$$

Equation (A30) is a desirable result of equation (67).

Here, we also log-linearize the money demand functions. Combining equation (A27) and (A28) with (A29), respectively, money demand functions for currency *H* and *F* are

$$m_{H,t} - p_t = c_t - \left[\left(1 - \frac{\nu}{\theta} \right) \frac{\beta}{1-\beta} \{ \theta(1-\delta) + \nu \} \right] \hat{i}_{H,t} - \left(1 - \frac{\nu}{\theta} \right) \frac{\theta\beta}{1-\beta} \delta \hat{i}_{F,t} \tag{A31}$$

$$s_t + m_{F,t} - p_t = c_t - \left(1 - \frac{\nu}{\theta} \right) \frac{\beta}{1-\beta} (1-\delta) \hat{i}_{H,t} - \left\{ \left(1 - \frac{\nu}{\theta} \right) \frac{\theta\beta}{1-\beta} \delta + \frac{\nu\beta}{1-\beta} \right\} \hat{i}_{F,t} . \tag{A32}$$

Equations (A31) and (A32) mean that the relative size of the elasticity of substitution between consumption and currency index θ, and the elasticity of currency substitution between currency *H* and *F* ν determines the semi-elasticity of real money demand with respect to interest rates $\hat{i}_{H,t}$ and $\hat{i}_{F,t}$. A parameter θ measures liquidity effects, while ν measures currency substitution effects. If $\theta > \nu$ is satisfied, namely, the liquidity effects dominate the currency substitution effects, then a rise in $\hat{i}_{H,t}$ decreases the real money demand for both currency *H* and *F*. The same argument can be applied for a rise in $\hat{i}_{F,t}$, and the opposite pattern emerges when $\theta < \nu$. Combining equations (A31) and (A32) yields relative money demand function,

$$m_{F,t} + s_t - m_{H,t} = \frac{\nu\beta}{1-\beta} (\hat{i}_{H,t} - \hat{i}_{F,t}) . \tag{A33}$$

From equation (A33), we can see that when $\theta > \nu$ and $\hat{i}_{H,t}$ increases by 1 percent point, then the relative demand increases by $\nu\beta/(1-\beta)$. This is because, the real money demand for currency *H* decreases by $(1-\nu/\theta)\beta/(1-\beta)\{\theta(1-\delta)+\nu\}$ (equation (A31)), while the real money demand for currency *F* decreases by $(1-\nu/\theta)\beta/(1-\beta)(1-\delta)$ (equation (A32).)

Appendix 3. Approximation of the current account

Using the definition of $P_{BF,t}$, equation (64) can be written as

$$\frac{S_t (B_{F,t} + M_{F,t})}{1 + i_{F,t}} - \frac{i_{F,t}}{1 + i_{F,t}} S_t M_{F,t} = S_{t-1} (B_{F,t-1} + M_{F,t-1}^{s,H}) \frac{S_t}{S_{t-1}} + P_{H,t} Y_t - P_t C_t . \tag{A34}$$

Inserting Euler equation (15) into equation (A34), and dividing both sides by $P_H Y$ to obtain

$$\beta E_t\left[\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\frac{U_{C,t+1}}{U_{C,t}}\right]\frac{S_t(B_{F,t}+M_{F,t})}{P_H Y}-\frac{i_{F,t}}{1+i_{F,t}}\frac{S_t M_{F,t}}{P_H Y}$$
$$=\frac{S_{t-1}(B_{F,t-1}+M_{F,t-1}^{s,H})}{P_H Y}\frac{S_t}{S_{t-1}}+\frac{P_{H,t}Y_t}{P_H Y}-\frac{P_t C_t}{P_H Y} \tag{A35}$$

where the second term of right hand side is approximately equal to zero. Therefore, log-linear approximation of equation (A35) is

$$\beta E_t\left[1+p_t-p_{t+1}+s_{t+1}-s_t+u_{C,t+1}-u_{C,t}\right]nfa_t=nfa_{t-1}(1+s_t-s_{t-1})+p_{H,t}+y_t-p_t-c_t \tag{A36}$$

where we use the fact that $P_H Y = PC$ are satisfied in the initial symmetric steady state. (A36) can be approximated so as to yield a desirable equation

$$\beta nfa_t = nfa_{t-1}+p_{H,t}+y_t-p_t-c_t\,. \tag{A37}$$

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NOTES

1. The term dollarization refers to the use of a foreign currency in any of the three traditional functions of money: unit of account (price substitution), medium of exchange (currency substitution) and store of value (asset substitution).
2. See Giovannini and Turtelboom (1994), Calvo and Végh (1992, 1996) and Végh (2013) for surveys.
3. The small open economy model with incomplete financial markets possesses a steady state which depends on initial conditions and in which consumption and net foreign assets follow a non-stationary process. In this case, further modifications are required. They include (1) a model with endogenous discount factor, (2) a model with a debt-elastic interest rate premium, (3) a model with convex portfolio adjustment costs, (4) a model with non-Ricardian agents and a demographic structure *a la* Blanchard-Yaari-Weil. See Schmitt-Grohé and Uribe (2003) and Lubik (2007).
4. For simplicity, we will ignore the foreign household's profit from selling the bond F to home household, $\frac{B_{F,t}}{1+i_{F,t}^*}\left(\frac{1}{\Psi(\cdot)}-1\right)$.
5. As mentioned above, if utility function is separable with respect to C_t and Z_t, or $1/\theta=\sigma$ is satisfied, money becomes neutral. Otherwise, such like in our model, output level is no longer invariant to monetary policy, which generates a gap in output. This gap is increasing in both home and foreign nominal interest rates. To achieve efficient allocation, Woodford (2003) assumes that central bank can pay interest on the holdings of home currency and that it can tax the holdings of foreign currency. This allocation is also achieved by simply setting $\hat{i}_{H,t}=\hat{i}_{F,t}=0$.
6. In addition to complete financial market and flexible price, we suppose the situation without currency substitution, namely, $M_{F,t}=0$ for all t.

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