

**Re-evaluating Japan's Quantitative Easing Policy (2001–2006):  
An Application of the TVP-VAR Model.**

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**ABSTRACT**

This study is the first to examine the changes in the effectiveness of Japan's quantitative easing policy (QEP) at a monthly frequency, using time-varying parameter VAR estimation. The results are significantly different from those obtained in previous studies. First, it is shown that the QEP's effects on the real economy have varied over time. The QEP effectively influenced production through most of the observation period, but its exact impacts varied throughout. However, with some exceptions, the QEP had limited effects on prices. Second, the QEP influenced stock price significantly all through the observation period. This result suggests that the stock price channel, might have been the effective transmission channel for the QEP.

**Keywords:** Quantitative easing policy (QEP), Bank of Japan (BoJ), Time-Varying Parameter vector autoregressive (TVP-VAR) model, monetary policy, transmission mechanism.

**JEL Classification:** E44, E52, E58

## **1. Introduction**

Many advanced countries, including Japan, adopted short-term interest rates as operating targets in the 1990s. However, the central bank of each country experienced the so-called zero bound problem, in which banks could not lower short-term nominal interest rates any further. Japan faced the zero limit in the late 1990s, earlier than other countries. Therefore, Japan's quantitative easing policy (QEP), implemented by the Bank of Japan (BoJ) from March 2001 to March 2006, deserves thorough examination.

As shown in Table 1, although researchers have extensively examined the effects of the QEP, they have not yet reached a consensus. Many previous studies used fixed parameter estimation and evaluated the QEP without considering whether its effectiveness changed over time. Because it was a totally new policy regime and the financial structure of the Japanese economy was changing radically, however, the QEP's effectiveness is likely to have varied over time. By adopting a time-varying parameter vector autoregressive (TVP-VAR) model, this study verifies that the effectiveness of the QEP varied throughout its implementation. Specifically, I apply the TVP-VAR model to monthly data and use production levels, prices, the BoJ's current account balance (CAB) and stock price as variables for the estimation. This study's results will offer an accurate basis for exploring future directions for monetary policy in advanced countries, including Japan.

The main findings are as follows. First, the QEP's effects on the real economy have varied over time. The QEP effectively influenced production throughout most of the observation period, but its exact impacts varied. However, the QEP had limited effects on prices, with some exceptions. Second, the QEP influenced stock price significantly all through the observation period. This result suggests that the stock price channel, might have been the effective transmission channel for the QEP. This finding is similar to the suggestion of Honda et al. (2013).

The remainder of this paper is structured as follows. Section 2 reviews previous studies, after which the TVP-VAR model is explained in Section 3. In Section 4, an outline is then provided of the estimation technique. Estimated results are explained in Section 5, with the conclusions presented in Section 6.

## **2. Background**

In the early 1990s, following the collapse of its financial bubble, Japan experienced a long period of economic stagnation. Consequently, the BoJ introduced a zero interest rate policy in February 1999 and reduced the call rate, which was the operating target, to a level close

to zero. Moreover, it introduced the QEP in March 2001 as an additional step of monetary easing. The defining new characteristic of this policy was that the operating target was changed from the call rate to the BoJ's CAB.<sup>1</sup> The BoJ raised the CAB target repeatedly and supplied funds to the market by purchasing Japanese government bonds from banks. In addition, the BoJ declared that it would continue its QEP until the inflation rate became positive and stable. In March 2006, the BoJ announced the termination of the QEP.

The QEP has certain unique characteristics that have been neglected by previous studies. First, it was a new policy regime when it was introduced. As such, its potential effects were not known at the time of its implementation. Evaluations of its effectiveness evolved through time in the financial markets, and its impacts on asset prices are thus expected to have changed accordingly. Second, during the QEP implementation, the Japanese government changed its prudence policy and forced financial institutions to actively dispose of bad loans. At the same time, corporations tried to reduce their debts levels. Thus, the QEP's impacts on bank lending are also expected to have changed over time. In sum, financial structures were changing substantially during the QEP implementation period, and the QEP's effectiveness is consequently expected to have varied significantly. This study focuses on these points.

By neglecting these important points, previous studies of the QEP suffer from two problems.<sup>2</sup> The first problem is that most of the previous studies estimated parameters that were fixed during the QEP implementation period.<sup>3</sup> In contrast, the present study employs the TVP-VAR model, established by Primiceri (2005), in order to measure the changes in the effectiveness of the QEP and to capture the structural changes in the economy.

Prior studies that analyzed the QEP using the TVP model captured the aforementioned structural changes.<sup>4</sup> However, these examinations suffer from a second problem, which stems from the frequency of the data employed in the studies. The BoJ revised its CAB target at its monthly policy committee meetings. In effect, the CAB target was often adjusted on a monthly basis.<sup>5</sup> Thus, monthly data should be used to evaluate the changing effectiveness of the QEP policy actions. However, all of the previous TVP studies used quarterly data. This study employs monthly data to perform a more detailed and accurate analysis of the QEP with the TVP-VAR model and thus to re-evaluate the effectiveness of the QEP. This is done in order to avoid the aforementioned two problems. Following Honda et al. (2013), who completed a detailed evaluation of the QEP's effects, this study uses a three-variable base model that includes the industrial production index, the core consumer price index, and the

BoJ's CAB. Further, in order to examine the transmission mechanisms of the QEP, the stock price is added to the base model.<sup>6</sup>

### 3. The TVP-VAR model

This section introduces the TVP-VAR model. The model is employed in a manner similar to that in, Nakajima (2011), Nakajima & Watanabe (2011) and Primiceri (2005). The model is formulated as follows:

$$A_t y_t = C_{1t} y_{t-1} + C_{2t} y_{t-2}, \dots + C_{st} y_{t-s} + \epsilon_t, \\ \epsilon_t \sim N(0, V_t), \quad t = s + 1, s + 2, \dots, T,$$

where  $y_t$  is a vector of economic variables ( $n \times 1$ );  $A_t$  and  $C_{it}$  are matrices of time-varying coefficients ( $n \times n$ ) ( $i = 1, 2, \dots, s$ );  $\epsilon_t$  is a vector of the structural shocks ( $n \times 1$ ); and  $V_t$  is a variance-covariance matrix ( $n \times n$ ).<sup>7</sup> The reduced form of this model is then

$$y_t = B_{1t} y_{t-1} + B_{2t} y_{t-2}, \dots + B_{st} y_{t-s} + u_t, \\ u_t \sim N(0, A_t^{-1} V_t A_t^{-1'}),$$

where  $B_{it} = A_t^{-1} C_{it}$ ,  $u_t = A_t^{-1} \epsilon_t$ .  $u_t$  is an error term vector ( $n \times 1$ ). Then, regarding the variance of  $u_t$ , I perform a Cholesky decomposition and impose recursive restriction,

$$A_t^{-1} V_t A_t^{-1'} = A_t^{-1} \Sigma_t \Sigma_t' A_t^{-1'},$$

where  $A_t$  is a lower triangular matrix in which the diagonal elements are equal to one, and  $\Sigma_t$  is the diagonal matrix. I then define  $\beta_t = \text{vec}[B'_{1t}, \dots, B'_{st}]$  and  $X_t = I_s \otimes (y'_{t-1}, \dots, y'_{t-s})$ ,

$$y_t = X_t \beta_t + A_t^{-1} \Sigma_t e_t, \\ e_t \sim N(0, 1),$$

where  $u_t = A_t^{-1} \epsilon_t = A_t^{-1} \Sigma_t e_t$ .<sup>8</sup> Here, I define the lower triangular elements of  $A_t$  as  $a_t = (a_{21,t}, a_{31,t}, a_{32,t}, \dots, a_{nn-1,t})'$  and the natural logarithm for diagonal elements of  $\Sigma_t$  as  $h_t = (h_{11,t}, \dots, h_{nn,t})'$ . From the above considerations, the time-varying parameters of this model are  $(\beta_t, a_t, h_t)$ . Then, the dynamics of these parameters are specified as follows:

$$\beta_{t+1} = \beta_t + u_t^\beta, \\ a_{t+1} = a_t + u_t^a, \\ h_{t+1} = h_t + u_t^h.$$

Moreover, the error term vector of each of the variables is

$$\begin{pmatrix} u_t^\beta \\ u_t^a \\ u_t^h \end{pmatrix} \sim N \left( 0, \begin{pmatrix} w_\beta & 0 & 0 \\ 0 & w_a & 0 \\ 0 & 0 & w_h \end{pmatrix} \right),$$

where it is assumed that  $(w_\beta, w_a, w_h)$  are diagonal matrices.<sup>9</sup> The next section provides an outline of the technique used for estimating this model.

## 4. Methods

### 4.1. Data

This study uses monthly data from April 1998 to March 2008.<sup>10</sup> The variables include the index of industrial production ( $y$ ), the consumer price index ( $p$ ), the BoJ CAB ( $m$ ) and the Nikkei stock average ( $s$ ).<sup>11</sup> This study estimates two forms of the model: the basic model ( $y, p, m$ ); the stock price model ( $y, p, m, s$ ).<sup>12</sup>

### 4.2. Bayesian estimation

This section presents the process for estimating the models.<sup>13</sup> The estimation of the TVP-VAR model is described in detail in Nakajima (2011) and Nakajima & Watanabe (2011). The present study conducts a Bayesian estimation using the Markov chain Monte Carlo (MCMC) method based on Nakajima (2011). First,  $y = \{y_t\}_{t=s+1}^T, \beta = \{\beta_t\}_{t=s+1}^T, h = \{h_t\}_{t=s+1}^T$  and  $w = (w_\beta, w_a, w_h)$  are defined. Moreover, a sample is obtained from the posterior probability density function  $\pi(\beta, a, h, w|y)$  by the following order based on the data and prior probability density function of each parameter. An initial sample of 30,000 is generated; then, it is discarded and another sample of 30,000 generated. Next, the sampling frequency is defines as  $j = 1, 2, \dots, 30,000$ . The steps of this process are as follows:

1. Set initial values of  $\beta^0, a^0, h^0, w^0$
2. Sample  $\beta^{j+1}$  from  $\pi(\beta|a^j, h^j, w_\beta^j, y)$
3. Sample  $a^{j+1}$  from  $\pi(a|\beta^{j+1}, h^j, w_a^j, y)$
4. Sample  $h^{j+1}$  from  $\pi(h|\beta^{j+1}, a^{j+1}, h^j, w_h^j, y)$
5. Sample  $w_\beta^{j+1}$  from  $\pi(w_\beta|\beta^{j+1})$
6. Sample  $w_a^{j+1}$  from  $\pi(w_a|a^{j+1})$
7. Sample  $w_h^{j+1}$  from  $\pi(w_h|h^{j+1})$
8. Perform sampling repeatedly from step 2 to step 7 until  $j=30,000$ .

The initial state of the time-varying parameters is assumed as follows:

$$\begin{aligned}\beta_0 &\sim N(0, 10I), \\ a_0 &\sim N(0, 10I), \\ h_0 &\sim N(0, 10I),\end{aligned}$$

where  $\tilde{w}_{\beta_k}^2, \tilde{w}_{a_k}^2, \tilde{w}_{h_k}^2$  are  $k$ -th diagonal elements of  $w_{\beta}, w_a, w_h$ . The priors of the basic model and stock price model are assumed as follows:

$$\tilde{w}_{\beta_k}^2 \sim IG(50, 0.001),$$

$$\tilde{w}_{a_k}^2 \sim IG(5, 0.001),$$

$$\tilde{w}_{h_k}^2 \sim IG(5, 0.001).$$

Two lags are set in each model.<sup>14</sup> The estimated results are given in the following section.<sup>15</sup>

## 5. Estimation results

The changes in the impulse responses for each model are shown in Figure 1 and Figure 2. These impulse responses change over time since the estimated parameters change over time in the TVP-VAR estimation. The following section is solely focused on the QEP implementation period and examines the impulse responses in each model.

Figure 1 illustrates the responses to the monetary policy shock in the basic model. Since the QEP is implemented as an easing policy, it is expected that production will respond positively. The empirical results conform to this expectation, but the size of the impulse responses varies across sub-periods. The first sub-period is from the start of QEP implementation to the end of 2002, the second is from early 2003 to mid-2004, and the third runs from mid-2004 until the end of QEP. The magnitude of the impulse responses in the first sub-period was larger than those in the second and third sub-periods. These are among the most interesting and unique findings based on the TVP-VAR estimation applied in this study. Positive responses were expected for prices, as well. However, the results show that the price responses are relatively small and unstable in most sub-periods. This may partially reflect the price puzzle. The BoJ's CAB shows significant positive responses to the monetary policy shock throughout the period of QEP implementation, with its significance level gradually decreasing. This result is as expected and can be considered as a natural response with a lag. Therefore, this study focuses on the responses of production to the QEP in the following analysis.

Figure 2 illustrates the impulse responses of the stock price model. As in the basic model, production shows significant positive responses in all sub-periods starting from 2 months afterwards. Throughout the QEP period, the responses of stock price are generally positive and significant 1 month later. Moreover, stock price responds more quickly than does production. These results suggest that the stock price channel might have contributed to production responses throughout the QEP period.

## 6. Conclusions

The results obtained in this study are substantially different from those of previous studies. Most previous studies, using fixed parameter estimation, evaluated QEP without considering the dynamic nature of its effectiveness. Moreover, although some previous studies used TVP-VAR estimation, they failed to consider substantial monthly changes in the QEP implementation and in the economic structure, by making use of quarterly data only. This study is the first examination to evaluate changes in policy effectiveness at a monthly frequency. In addition, previous studies that also made use of the TVP-VAR model focused on certain time points during the QEP period and did not examine changes in policy effectiveness over time. Consequently, the present study is the first to reveal changes in effectiveness spanning the entire QEP period.

The main findings are as follows. First, QEP's effects on the real economy varied over time. QEP effectively influenced production throughout the QEP period, but the size of the effect was largest from the start of QEP implementation until late 2002 (the first sub-period). On the other hand, it had limited effects on prices through the QEP. Second, the QEP influenced stock price significantly all through the observation period. This result suggests that the stock price channel, might have been the effective transmission channel for the QEP. These findings are critically important for any countries implementing monetary easing policies under a zero interest rate regime.

## 7. Appendix

Figure 3 and 4 show the sample autocorrelations of samples generated in the basic model and stock price model. They illustrate that the autocorrelation of each parameter attenuates sufficiently, indicating that the sampling method efficiently produces samples with low autocorrelation. Furthermore, in Table 2, I confirm whether the sample converges sufficiently in the posterior probability density function and present Geweke (1992)'s convergence diagnostics (CD) for a number of parameters for each model. In Table 2, I express the  $p$ -value of the CD statistics under the null hypothesis that the sample converges in the posterior distribution of the parameter in each model. The hypothesis cannot be rejected at the 10% significance level. These results suggest that the estimated samples for each model are efficiently generated.

## ACKNOWLEDGEMENTS

I am grateful to Toshiki Jinushi (Kobe University), Yoichi Matsubayashi (Kobe University), Kenneth N. Kuttner (Williams College), Masahiko Sibamoto (Kobe University), Nobuyuki Oda (Kyourin University), and Koichi Yano (Komazawa University) for valuable comments and suggestions. This is a revised version of a paper presented at Rokko Forum, Japan Society of Monetary Economics and Japanese Economic Association.

## NOTES

1. For more details about the QEP, please see Honda et al. (2013).
2. Numerous studies of the QEP have been undertaken, but the present study focuses on those using VAR models in order to consider the macroeconomic effects.
3. Representative previous researches that has estimated fixed parameters are Honda et al. (2013), Iwata (2010), Kamada & Sugo (2006), Kimura et al. (2003), Schenkelberg and Watzka (2013). On the other hand, Fujiwara (2006), Inoue & Okimoto (2008) and Girardin & Moussa (2010) adopted a Markov-switching (MS) VAR model. They find that one of the regimes prevails throughout most of the QE period, and thus I could consider the VAR parameters to be constant during the QE period even if the MS-VAR model is estimated. Furthermore, Iwata & Wu (2006) is representative of research on zero interest rate policy implementation, and Miyao (2002) offers an example before the zero interest rate policy and the QEP.
4. Representative examples of the research that has analyzed the QEP using the TVP model are Franta (2011), Kimura & Nakajima (2013), Michaelis & Watzka (2014), Nakajima et al. (2011) and Moussa (2010). In Kimura et al. (2003), the coefficient matrices of the VAR model are time varying, but the variances of the structural shock fixed across time.
5. During the QEP period, the target figure of the BoJ's CAB was changed nine times, out of which three instances were due to the previous changes occurring within the quarter.



6. Honda et al. (2013) suggests that stock price channel is effective through the whole period of QEP implementation.
7.  $A_t$  indicates the simultaneous relations among the economic variables.
8.  $I$  is an identity matrix.
9. The dimensions of  $w_\beta, w_a$  and  $w_h$  are  $(n^2s \times n^2s)$ ,  $((n^2 - n)/2 \times (n^2 - n)/2)$  and  $(n \times n)$ .
10. All data are in log form and de-meaned. I followed Honda et al. (2013) in using the level of variables rather than first-difference. They justified this approach based on the consistency of the estimated parameters and the richness of the contained information.
11. This study uses the consumer price index except for fresh foods (core CPI). The Nikkei Stock Average is an end-of-month value. Regarding the sources of the data, the prices were originally obtained from the Statistics Bureau and the Ministry of Internal Affairs and Communications; other data were sourced from Datastream. In addition, these data use the index of industrial production and core CPI, seasonally adjusted. The CAB is seasonally adjusted using X-12 ARIMA (Eviews).
12. The orders of variables are specifically described.
13. This study used the TVP-VAR model (Matlab) as given in Nakajima (2011) to estimate each parameter. In addition, the package was modified to simulate the impulse responses.
14. In impulse response analysis, the estimated results when setting three lags are similar to those using two lags.
15. The stability of the estimated results is discussed in the Appendix.

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Table 1. QEP evaluation

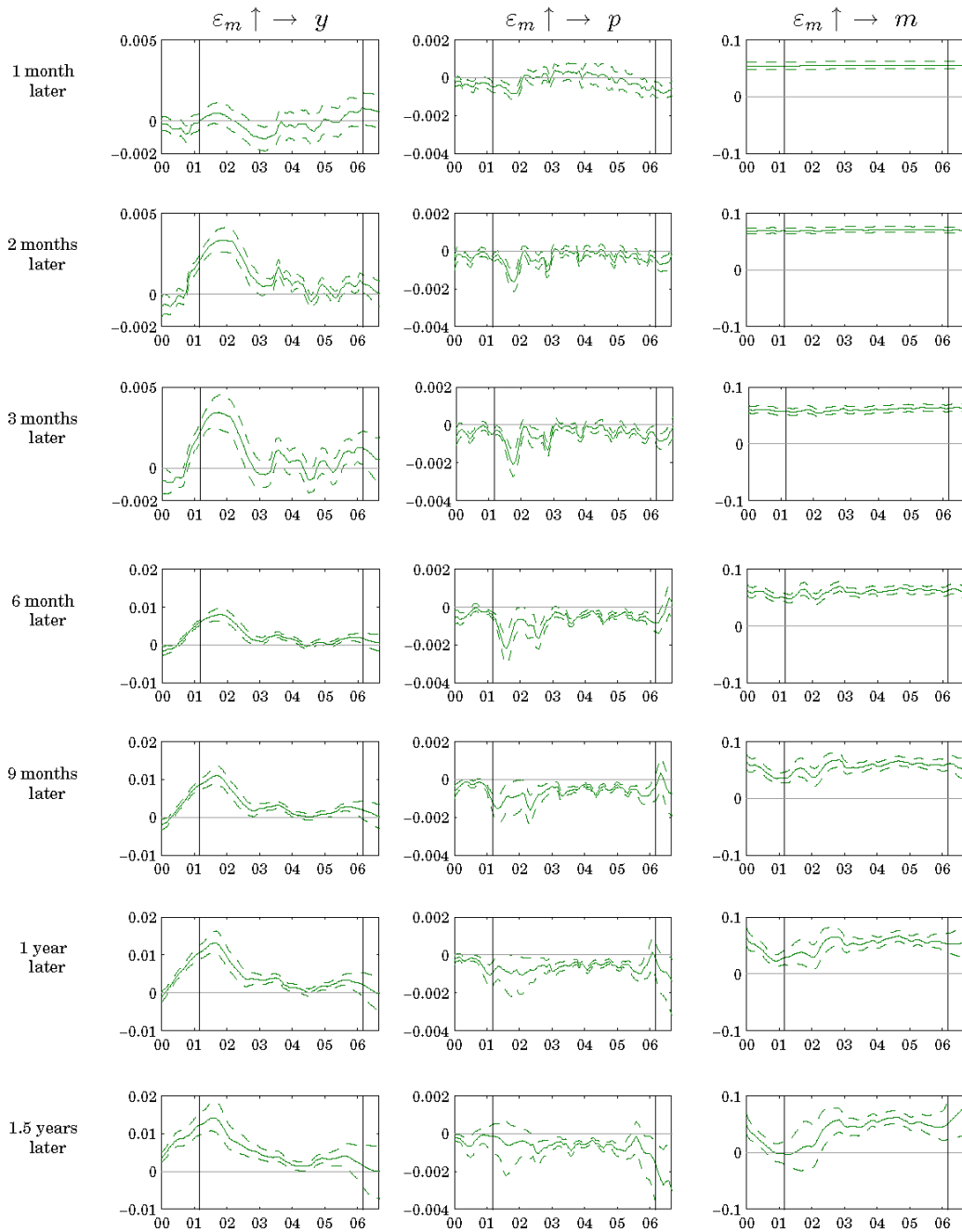
	Production (significance)	Prices (significance)
Schenkelberg and Watzka (2013)	limited (partial)	effective (exist)
Girardin and Moussa (2011)	effective (exist)	effective (exist)
Iwata (2010)	effective (exist)	limited (nothing)
Honda et al. (2013)	effective (exist)	limited (nothing)
Kamada and Sugo (2006)	limited	limited
Fujiwara (2006)	limited (partial)	limited (partial)
Kimura et al. (2003)	limited	limited
Inoue and Okimoto (2008)	effective (exist)	limited (partial)
Franta (2011)	effective (exist)	effective (exist)
Nakajima et al. (2011)	limited (nothing)	
Kimura and Nakajima (2013)	limited (nothing)	limited (nothing)
Michaelis and Watzka (2014)	limited (nothing)	effective (exist)
Moussa (2010)	effective (exist)	effective (exist)

Table 2. CD statistics ( $p$ -value)

Parameter	$\beta$	$a$	$h$	$w_\beta$	$w_a$	$w_h$
Basic model	0.495	0.273	0.508	0.266	0.292	0.339
Stock price model	0.724	0.204	0.158	0.575	0.992	0.456

$(\beta, a, h)$  are element (1, 1) of each parameter in November 1999. Moreover,  $(w_\beta, w_a, w_h)$  are the elements (1, 1) of each parameter.

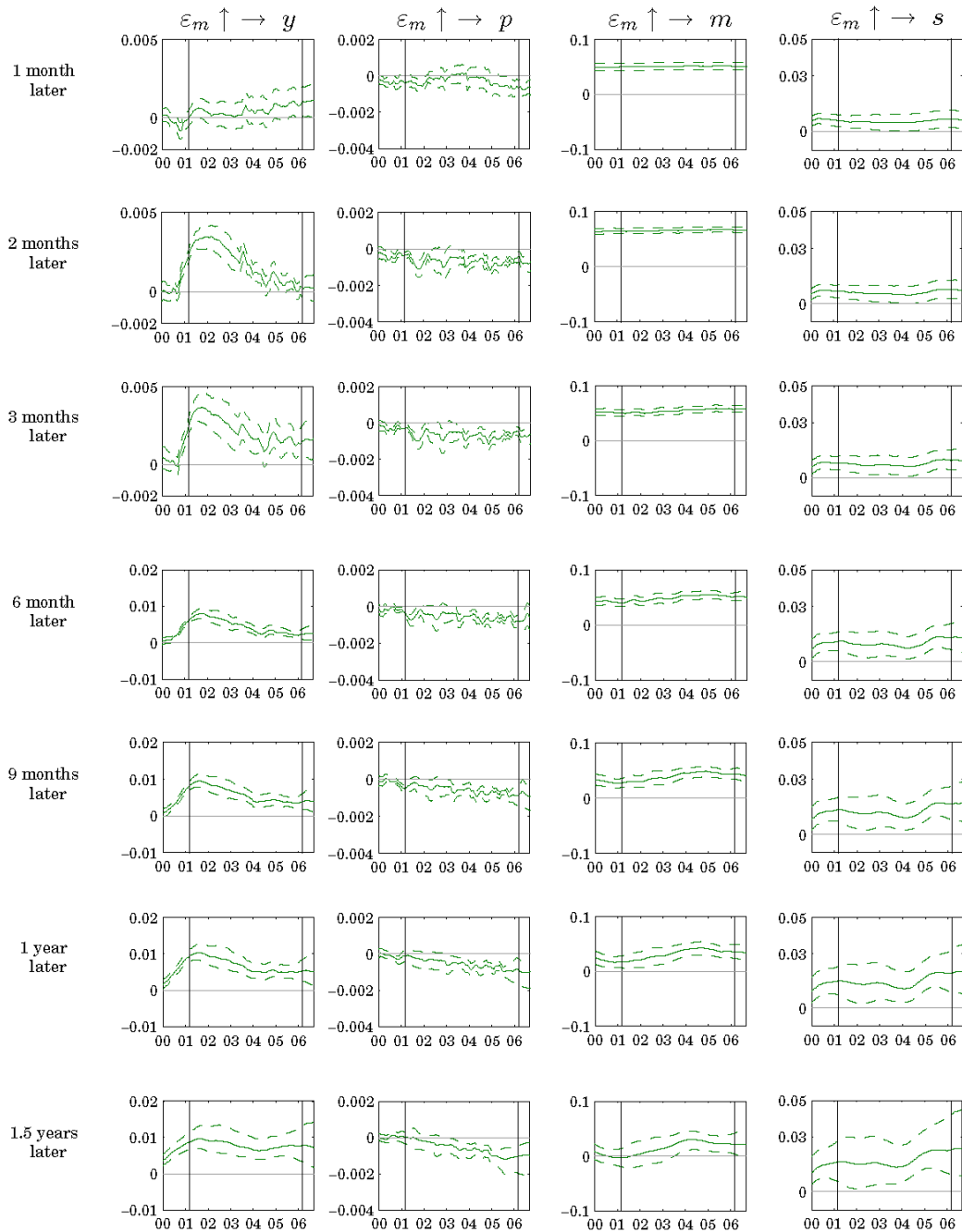
Figure 1. The impulse responses of the basic model



The figure illustrates the responses of each variable (columns) to the monetary policy shock at specific periods after the shocks (rows): 1, 2, 3, 6, and 9 months and 1 and 1.5 years.  $\epsilon_m \uparrow \rightarrow x$  is the impulse response of variable ( $x$ ) to the monetary policy shock. The horizontal axis represents the time period from January 2000 to September 2006; the impulse

responses are calculated with parameters estimated for each point in time. The vertical axis expresses the size of the response. Based on 30, 000 samples, the solid lines indicate the posterior medians of the impulse responses, and the dashed lines represent the 25th and 75th percentiles, indicating the significant influences, as in Nakajima & Watanabe (2011). The two solid vertical lines show the starting and ending dates of QEP implementation (March 2001 and March 2006, respectively). The monetary policy shock is represented by one standard error of the estimated structural shocks, averaged over all the periods in each model.

Figure 2. The impulse responses of the stock price model

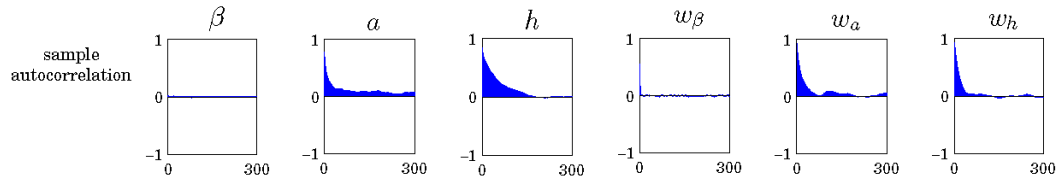


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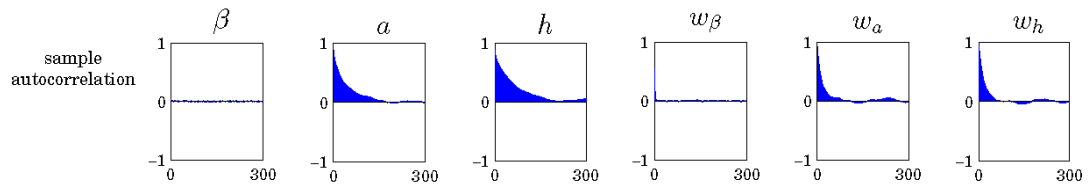


Figure 3. Estimation results of the basic model for selected parameters



$(\beta, a, h)$  are element (1, 1) of each parameter in November 1999. Moreover,  $(w_\beta, w_a, w_h)$  are the elements (1, 1) of each parameter. The vertical line shows the autocorrelation function, and the transverse axis shows sampling frequency (300 of 30,000 samples).

Figure 4. Estimation results of the stock price model for selected parameter



$(\beta, a, h)$  are element (1, 1) of each parameter in November 1999. Moreover,  $(w_\beta, w_a, w_h)$  are the elements (1, 1) of each parameter. The vertical line shows the autocorrelation function, and the transverse axis shows sampling frequency (300 of 30,000 samples).