The liquidity effect and tightening effect of the zero lower bound

Kohei Hasui
Kobe University
2-1 Rokkodai-cho Nada-ku, Kobe, 657-8501, Japan
Email: 100e107e@stu.kobe-u.ac.jp

ABSTRACT

The tightening effect of the zero lower bound on the nominal interest rate is a non-trivial topic in monetary policy: at the zero lower bound, responding to a rise in money growth by reducing the nominal interest rate—what is called the liquidity effect—is not possible because the nominal interest rate cannot be further decreased. However, the absence of the liquidity effect caused by the zero lower bound might amplify the tightening effect of the zero lower bound. I call this tightening effect of the zero lower bound through its liquidity effect on the economy the *rebound of liquidity effect*, and demonstrate it quantitatively with a simple dynamic stochastic general equilibrium framework.

1. Introduction

In the short run, a rise in the growth of nominal money lowers the short-run nominal interest rate. This is called the *liquidity effect*. The liquidity effect does not appear when the nominal interest rate hits the zero lower bound, which is the situation that developed countries faced after the colossal financial crisis of 2008. However, the absence of the liquidity effect caused by the zero lower bound might affect the entire economy through its tightening effect. In this paper I describe how I examined the tightening effect of the zero lower bound on the economy quantitatively in a dynamic stochastic general equilibrium (DSGE) framework that generates a strong liquidity effect.

Researchers have studied the liquidity effect both theoretically and empirically. Since standard DSGE models do not generate the liquidity effect,[1] researchers have retrofitted the structures of various models to generate the effect (Christiano, 1991; Christiano and Eichenbaum, 1995; Christiano and Gust, 1999; Christiano et al., 1997, 2005; Keen, 2004; Edge, 2007). Christiano (1991), Christiano and Eichenbaum (1995), and Christiano and Gust (1999) modeled the liquidity effect by introducing a structure in which the timing of loan market participation between households and firms differed (the limited participation model). In the empirical literature, analyses were performed mainly with vector autoregression (VAR) (Christiano et al., 1999; Leeper and Gordon, 1992; Leeper et al., 1996). Since the liquidity effect does not appear in the straightforward specification of VAR, Christiano et al. (1999), for example, used the non-borrowed reserve corresponding to the variable of monetary policy to generate the liquidity effect. However, these previous studies examined the liquidity effect without considering the zero lower bound on nominal interest rates. Since the liquidity effect does not appear when the nominal interest rate hits the zero lower bound, it has not been studied in situations involving the zero lower bound.

It is possible, however, that the zero lower bound on nominal interest rates could have a tightening effect through the liquidity effect. If the model economy has a structure which generates a strong reduction of the nominal interest rate in response to increases of money growth, a large part of the nominal interest rate which could decrease if the zero lower bound does not exist might spill over into another part of the economy, such as the output, inflation, and real money, etc., because the non-negativity constraint does not allow the nominal interest rate to decrease further. Thus the tightening effect of the zero lower bound could depend on the strength of reductions in the nominal interest rate if the zero lower bound did not exist. This line of reasoning must be explored by a theoretical rather than empirical analysis. I borrow the specification of households' portfolio adjustment cost function developed by Christiano and

Gust (1999) to generate a persistent and strong liquidity effect. This specification enables the model to generate and adjust the strength of the liquidity effect easily, without a limited participation formation that includes complicated structures. I show that a positive money growth shock can stimulate the economy more with a strong liquidity effect than without the liquidity effect when the nominal interest rate does not hit the zero lower bound. I also show that a positive money growth shock cannot stimulate the economy more with a strong liquidity effect than without this effect when the nominal interest rate hits the zero lower bound. These results demonstrate that the tightening effect of the zero lower bound might be amplified by a structure which generates the strong liquidity effect. This possibility has not received much attention.

The remainder of this paper is structured as follows: Section 2 presents a description of the model, and Section 3 shows the results of the simulation. Section 4 contains my conclusions.

2. The model

This section describes the economic model. The model includes households, the firm sector and the government. The firm sector consists of intermediate goods producers and final goods producers; firms in the intermediate goods sector produce differentiated goods following the Calvo (1987) sticky pricing rule.

Optimization problems for households and firms derive the aggregate demand equation, the money demand equation and the aggregate supply equation. I describe details for the derivation of these model equations below.

2.1. Households

Households obtain utility from consumption C_t and real money balances m_t, and disutility from the labor supply N_t.

In addition, I introduce the portfolio adjustment cost function of real money balances into households' utility function in order to generate the strong liquidity effect.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{m_t^{1-\varsigma}}{1-\varsigma} - \frac{N_t^{1+\eta}}{1+\eta} - \Psi(m_t, m_{t-1}) \right] \tag{1}$$

where β denotes the subjective discount factor, σ denotes the relative risk aversion of consumption, ς denotes the relative risk aversion of real money, and η denotes Frisch elasticity of labor supply. The last term in the bracket is the adjustment cost function represented by Christiano and Gust (1999).

$$\Psi(m_t, m_{t-1}) = \delta \left\{ \exp\left[\alpha\left(\frac{m_t}{m_{t-1}} - 1 \right) \right] + \exp\left[-\alpha\left(\frac{m_t}{m_{t-1}} - 1 \right) \right] - 2 \right\}$$

$$\Psi(m,m) = 0,$$

where α and δ are non-negative parameters and m is the steady-state value of m_t. It is unusual to introduce the cost function into the utility. The reason for this assumption is (i) it provides a short-cut to generating a strong and persistent liquidity effect, and (ii) it is tractable to analyzing the impacts of the liquidity effect on the economy by adjusting parameters δ and α.[2]

The households' budget constraint is

$$C_t + m_t + b_t \le w_t N_t + R_{t-1}\frac{b_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} + \tau_t + d_t, \tag{2}$$

where b_t, w_t, R_t, Π_t, τ_t, and d_t are the real bond, real wage, gross nominal interest rate, gross inflation rate, lump-sum tax, and profit from the firm sector, respectively. Equation (2) shows that households allocate income from wages, bond yields, lump-sum tax and firms' profits to consumption, money and bonds. Households maximize lifetime utility (1) subject to Eq. (2) dynamically. The first-order conditions are as follows:

$$\frac{C_t^{-\sigma}}{R_t} = \beta E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}}, \tag{3}$$

$$m_t^{-\varsigma} - \Psi_1(m_t,m_{t-1}) - \beta E_t \Psi_2(m_{t+1},m_{t+1}) = (1 - (R_t)^{-1})C_t^{-\sigma}, \tag{4}$$

$$N_t^{\eta}C_t^{\sigma} = w_t, \tag{5}$$

where

$$\Psi_1(m_t,m_{t-1}) = \frac{\delta\alpha}{m_{t-1}}\left\{\exp\left[\alpha\left(\frac{m_t}{m_{t-1}} - 1\right)\right] - \exp\left[-\alpha\left(\frac{m_t}{m_{t-1}} - 1\right)\right]\right\}$$

$$\Psi_2(m_t,m_{t-1}) = -\frac{\delta\alpha m_t}{(m_{t-1})^2}\left\{\exp\left[\alpha\left(\frac{m_t}{m_{t-1}} - 1\right)\right] - \exp\left[-\alpha\left(\frac{m_t}{m_{t-1}} - 1\right)\right]\right\}.$$

Equation (3) is the Euler equation, which describes households' optimal intertemporal saving decisions. Equation (4) is the money demand equation, which describes the opportunity costs of holding money equal to the nominal interest rate. Equation (4) is different from standard money demand equations in that it depends on the expected real money balance and lagged real money balance. Since I assume the portfolio adjustment cost of the real money balance, the opportunity costs of holding money means that marginal adjustment costs are tacked onto the nominal interest rate. Finally, Equation (5) is the labor supply equation, which shows that the marginal disutility of labor must equal the real wage.

2.2. Final goods sector

The final goods sector inputs differentiated goods, $Y_t(i)$, $i \in (0,1)$, from the intermediate goods sector. The final goods sector then aggregates $Y_t(i)$ and sells it to households. The production function of the final goods sector is thus:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1,$$

where θ denotes the elasticity of substitution between different goods.

The intratemporal profit maximization of the final goods sector yields the goods for firm i's demand equation and aggregate price definition:

$$\frac{Y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t} \right)^{-\theta}, \tag{6}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{7}$$

2.3. Intermediate goods sector

Firms in the intermediate goods sector are an infinite continuum for the interval $(0,1)$, and firm i produces intermediate goods $Y_t(i)$ with the product function:

$$Y_t(i) = A_t N_t(i),$$

where A_t is productivity and is independent from the index i, and $A = 1$ at the steady state. By the cost minimization for firm i, the real marginal costs equal the real wage per unit of productivity:

$$s_t = \frac{w_t}{A_t}.$$

s_t denotes the real marginal cost, and firm i's real marginal product is independent from the index i.

The intermediate goods sector is monopolistically competitive. Moreover, each firm sets the prices for their goods following Calvo's (1983) pricing rule: the firm has a $1-\omega$ probability of changing the price, and a ω probability of not changing the price. Given these probabilities and the demand equation for goods i, intermediate goods firms set their optimal prices to maximize yields,

$$P_t^* = \left(\frac{\theta}{\theta-1} \right) \frac{E_t \sum_{k=0}^{\infty} \omega^k \Lambda_{t,t+k} s_{t+k} Y_{t+k}(i)}{E_t \sum_{k=0}^{\infty} \omega^k \Lambda_{t,t+k} Y_{t+k}(i)/P_{t+k}}, \tag{8}$$

where $\Lambda_{t,t+k}$ denotes the stochastic discount factor, $\beta^k (C_{t+k}/C_t)^{-\sigma}$. The intuition of Equation (8) is that the optimal price equals the expected marginal cost weighted by markup $\theta/(\theta-1)$.

Table 1 Baseline parameter values

β	subjective discount factor	0.99
σ	intertemporal substitution for consumption	2
ς	intertemporal substitution for real money	2
η	Frisch elasticity of labor	1
ω	price stickiness	0.8
α	parameter in cost function	2
δ	parameter in cost function	2
ρ	persistence of money growth	0.5

Applying Calvo's (1983) pricing rule to Equation (7), the aggregate price takes a recursive form, which consists of the changed price component and the unchanged price component.

$$P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{9}$$

All prices that cannot be changed in the current period are reduced into the second term on the right-hand side of Equation (9).

2.4. Government

The government has the following budget constraint:

$$R_{t-1}\frac{b_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} + \tau_t = m_t + b_t.$$

The monetary authority injects money into the system following the money growth rule.

$$\mu_t = \hat{m}_t - \hat{m}_{t-1} + \pi_t,$$

$$\mu_t = \rho\mu_{t-1} + \epsilon_t.$$

$\hat{m}_t$ denotes the percent deviation form of real money, π_t denotes the percent deviation form of inflation, μ_t is the growth rate of nominal money, and ϵ_t is an i.i.d. disturbance term.

3. Simulation

Now I present the complete log-linearized model.[3] All variables are the percent deviation from the steady state.

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) + u_t, \tag{10}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t, \tag{11}$$

$$\hat{m}_t + \psi\Delta\hat{m}_t - \psi\beta E_t\Delta\hat{m}_{t+1} = \frac{\sigma}{\varsigma}(x_t + y_t^f) - \frac{\beta}{(1-\beta)\varsigma}r_t, \tag{12}$$

$$\mu_t = \hat{m}_t - \hat{m}_{t-1} + \pi_t, \tag{13}$$

$$\mu_t = \rho\,\mu_{t-1} + \epsilon_t, \tag{14}$$

where $\psi = 2\delta\alpha^2/(\varsigma m^{1-\varsigma})$, $m = (1-\beta)^{-1/\varsigma}$, $y_t^f = (1+\eta)/(\eta+\sigma)\ln A_t$ is the flexible price output, $x_t = y_t - y_t^f$ is the output gap, and $u_t = E_t y_{t+1}^f - y_t^f$. If $\psi = 0$, the model becomes the standard New Keynesian model. Assume the baseline parameters are set as shown in Table 1; the parameters of the adjustment cost function, $\alpha = \delta = 2$, were set following Christiano and Gust (1999), and the parameters of the persistence of money growth, $\rho = 0.5$, were set following Christiano et al. (1998).[4,5] The simulation model was run and the results of impulse responses are analyzed below.

In Subsection 3.1, impulse responses of the model are derived with and without adjustment costs, and the occurrence of the strong liquidity effect is checked with the adjustment cost model. Subsection 3.2 derives impulse responses considering the zero lower bound on the nominal interest rate, and analyzes the tightening effect of the non-negativity constraint on the nominal interest rate in the model in which the strong liquidity effect occurred.

3.1. The liquidity effect

First, the assumption that the adjustment costs would generate the liquidity effect under the baseline parameters was confirmed. Figure 1 illustrates impulse responses to a positive money growth shock. The solid lines in the figure indicate impulse responses without adjustment costs, and the dashed lines are impulse responses with adjustment costs; the solid line in Figure 1e represents the money growth shock common to both models, with and without the adjustment cost. Figure 1 shows that the positive money growth shock stimulated the economy: the output-gap responded positively, and since the nominal rigidity of price levels existed, the inflation and real money balance increased, and the nominal interest rate decreased. There was a remarkable difference between the case with adjustment costs and the case without adjustment costs in terms of the output-gap and the nominal interest rate. The nominal interest rate decreased more when adjustment costs were present. This was the liquidity effect. Through the Euler equation for households, the strong liquidity effect raised the output gap more with adjustment costs than without. Thus, the monetary stimulus raised the economy to a greater extent through the strong liquidity effect.

3.2. Effect of the zero lower bound on the nominal interest rate

Next a non-negativity constraint was imposed on the nominal interest rate:
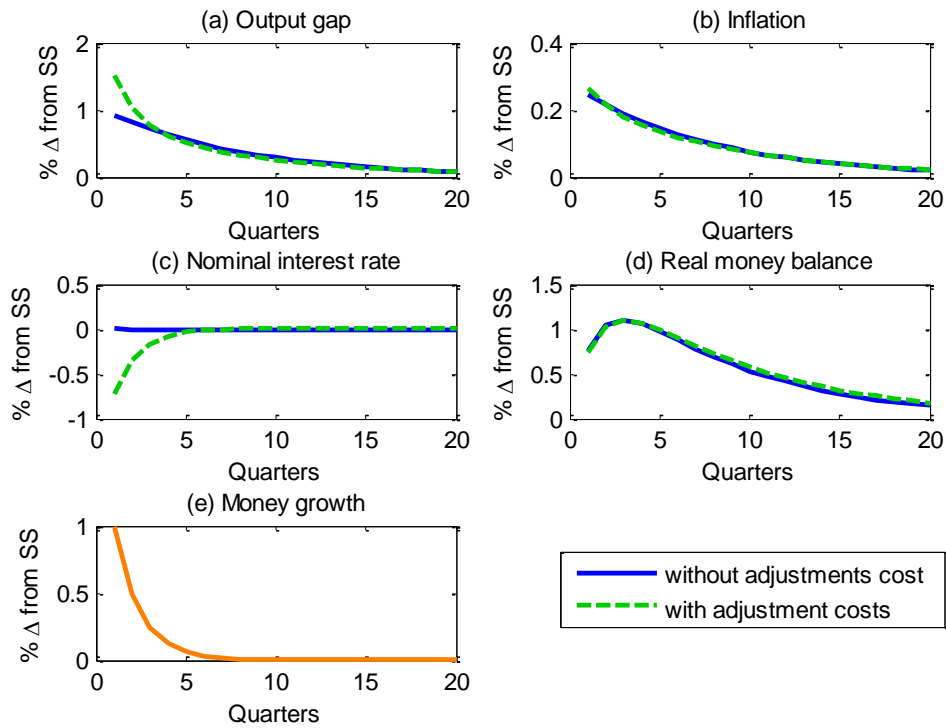
$$r_t \geq -\ln(1/\beta). \tag{15}$$

Figure 1. Impulse responses to a positive money growth shock. Solid lines: without the adjustment cost; dashed lines: with the adjustment cost

The zero lower bound in the log-linearized model became $-\ln(1/\beta)$, considering that R_t was $1/\beta$ at the steady state and that all the variables became zero at the steady state in the log-linearized model. The derivation of impulse response considering the zero lower bound is complicated because of its nonlinear nature. I used the MATLAB Toolbox provided by Guerrieri and Iacoviello (2014), which derives impulse responses while occasionally considering binding constraints by piecewise linear solutions.[6]

Figure 2 illustrates the impulse responses of the model with adjustment costs. Solid lines indicate responses with the constraint, and dashed lines represent the responses without the constraint. Figure 2 shows that the positive money growth shock stimulated the economy, and that the non-negativity constraint reduced its effect; the nominal interest rate hit the zero lower bound (Fig. 2c) and this contracted upward responses of output-gap and inflation (Fig. 2a,b). Since adjustment costs existed, the nominal interest rate decreased strongly toward the positive money growth shock. This allowed the nominal interest rate trend to hit the zero lower bound more intensely, with the potential to cause a more severe tightening effect on the economy. This effect is analyzed in Figure 3, which illustrates impulse responses to the positive money growth shock with the non-negativity constraint on the nominal interest rate. The solid lines in Figure
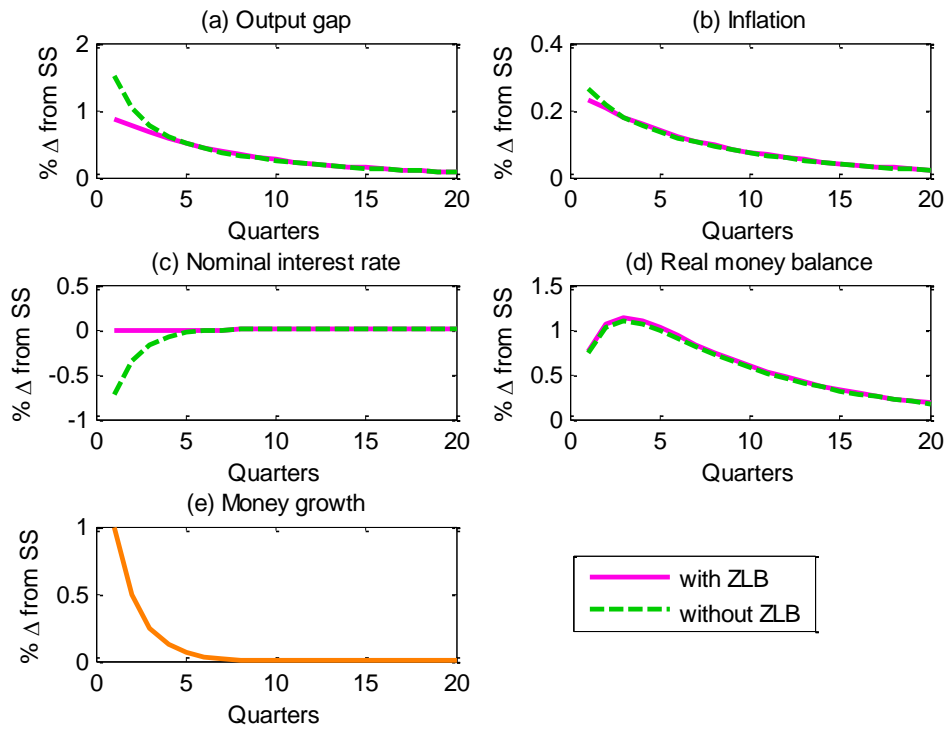
Figure 2. Impulse responses to a positive money growth shock. Solid lines: with the non-negativity constraint on the nominal interest rate; dashed lines: without the non-negativity constraint on the nominal interest rate.

3a–d indicate the case with adjustment costs, and the dashed lines indicate the case without adjustment costs. Figure 3e illustrates real money growth Δm_t (dashed line) and expected real money growth $E_t\Delta m_{t+1}$ (solid line) in cases in which adjustment costs and the zero lower bound exist. Figure 3f illustrates the difference between the discounted expectation of real money growth and the current real money growth in Figure 3e, $\beta E_t\Delta m_{t+1}-\Delta m_t$. The results presented in Figure 3a–d are the opposite of those shown in Figure 1. A money growth shock stimulates the economy less with adjustment costs than without adjustment costs — the solid lines of Figure 3a–d are below the dashed lines in output gap and inflation — but the difference is not as large as in the previous simulation (Fig. 3a,b). This implies that the zero lower bound might tighten the economy more through the strong liquidity effect. I call this the *rebound of liquidity effect*. The nominal interest rate hits the zero lower bound for six quarters and then rises higher than without adjustment costs. In Figure 3f, $\beta E_t\Delta m_{t+1}-\Delta m_t$, which is the term in the money demand equation, shows dynamics similar to the behavior of the nominal interest rate. Since the term $\psi(\beta E_t\Delta m_{t+1}-\Delta m_t)$ is derived from the adjustment cost specification, the behavior of the nominal
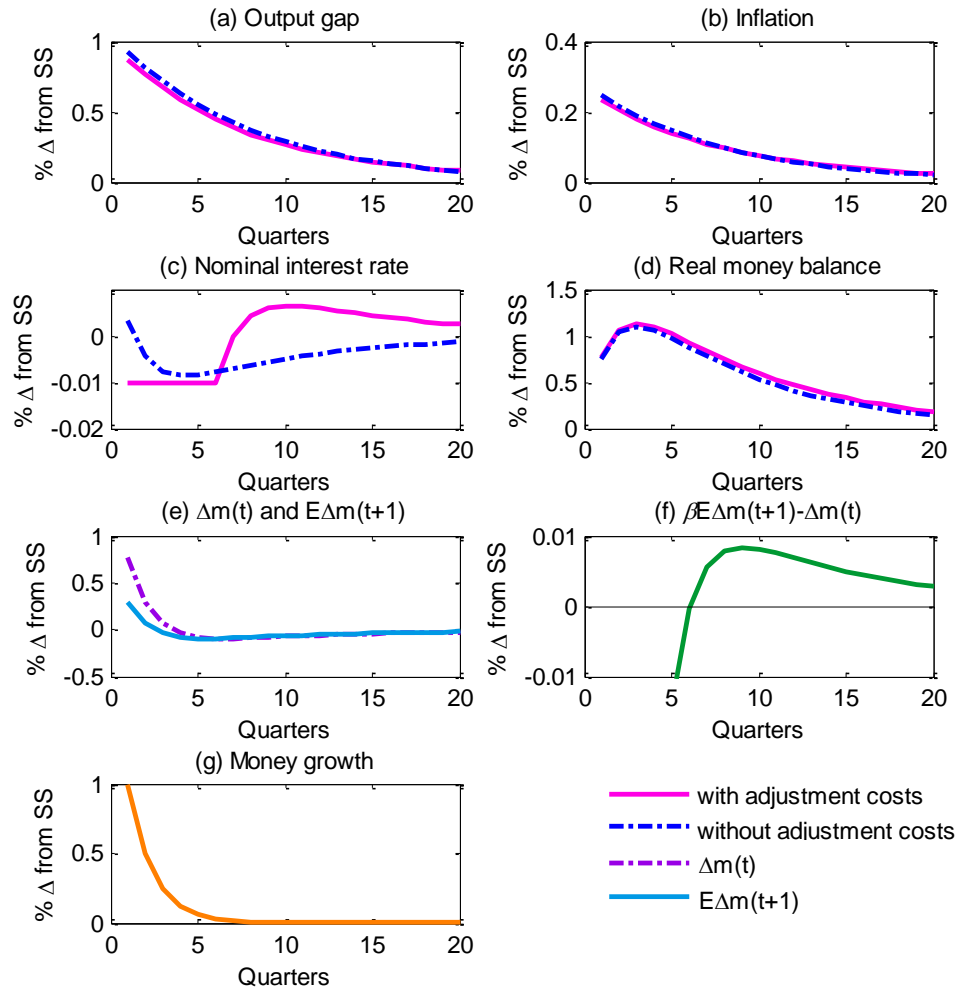
Figure 3. Impulse responses to a positive money growth shock with the non-negativity constraint on the nominal interest rate. a–d: Solid lines indicated cases with the adjustment cost; dashed lines indicate those without the adjustment cost. f: Expected real money growth minus real money growth with the adjustment cost (zoomed for interval [−0.01 0.01])
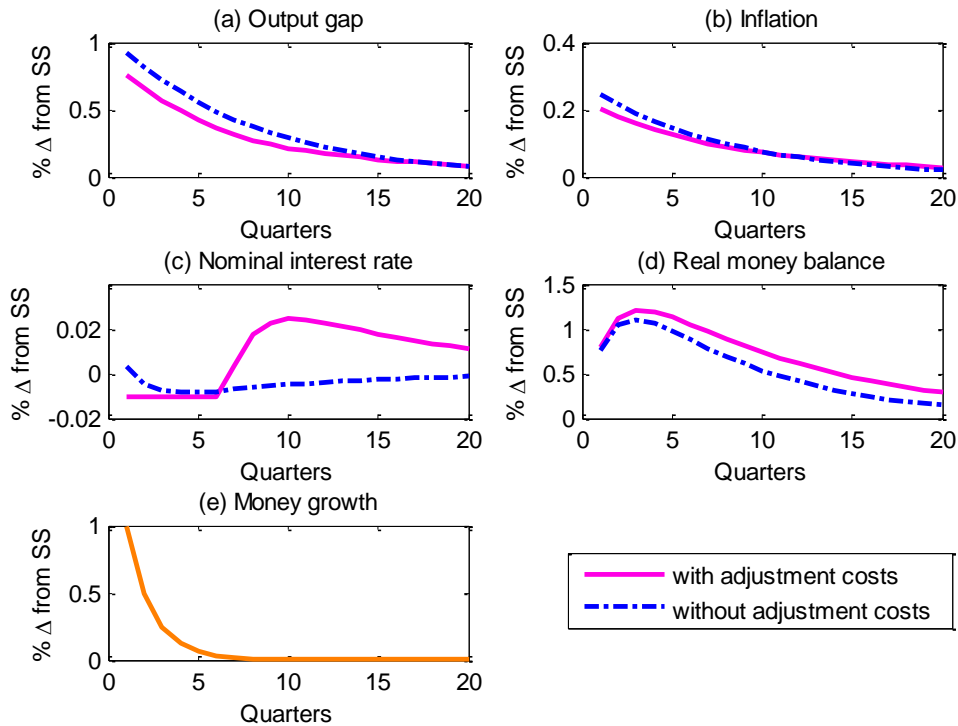
Figure 4. Impulse responses to a positive money growth shock with the non-negativity constraint on the nominal interest rate when $\alpha = \delta = 3$. Solid lines: with the adjustment cost; dashed lines: without the adjustment cost.

interest rate depends on the adjustment cost. Thus the liquidity effect occurs through adjustment costs, and the rebound of liquidity effect is expected to grow as adjustment costs increase. Let us now set the parameter $\alpha = \delta = 3$; Figure 4 illustrates the results with these parameters. As can be seen in the figure, the difference between the solid lines and dashed lines becomes large. The dynamics of the nominal interest rate are similar to the results shown in Figure 3 but are greater. Since the parameter $\alpha = \delta = 3$, the value of ψ increases. This increase amplifies the effect of $\beta E_t \Delta m_{t+1} - \Delta m_t$ to the nominal interest rate. According to the results shown in Figure 4, if the model has a structure which could generate the stronger liquidity effect when the zero lower bound does not exist, the great contraction effect occurs if the nominal interest rate hits the zero lower bound.

These results suggest some considerations for monetary policy. First, monetary policy can stimulate the economy by producing a strong liquidity effect when the nominal interest rate does not hit the zero lower bound. However, once the nominal interest rate hits the zero lower
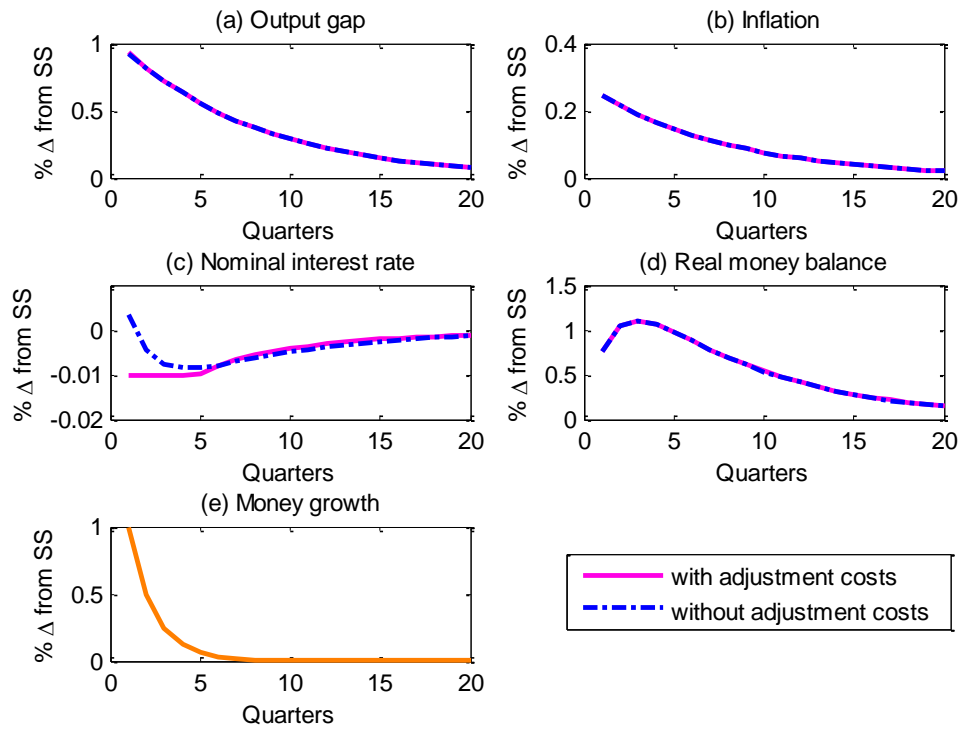
Figure 5. Impulse responses to a positive money growth shock with the non-negativity constraint on the nominal interest rate when $\alpha = \delta = 0.7937$. Solid lines: with the adjustment cost; dashed lines: without the adjustment cost.

bound, the existence of the strong liquidity effect might have the opposite outcome and amplify the tightening effect of the zero lower bound. Second, monetary policy should pay attention to the contraction effect of the zero lower bound when an economy that possesses the structures to generate a strong liquidity effect falls into the zero lower bound. It could then be important to develop policies to mitigate the structures of the strong liquidity effect. For example, a policy mitigating wage stickiness might be effective, because it would contribute to the dynamics of expected inflation, which puts upward pressure on the nominal interest rate through the Fisher relationship (Hasui, 2013). However, too much mitigation of inflation dynamics is risky, because once inflation diverges, the central bank cannot stop it.

3.3. Considering empirical evidence

It must be noted that the implications described in the previous subsection are based on theoretical and quantitative results. It is also important to consider empirical evidence. Nelson (2002) chose the parameter value $\alpha = \delta = 0.43$ considering the evidence analyzed by Taylor (1993) and Janssen (1998). Nelson (2002)'s choice of parameters implies $\alpha = \delta = 0.7937$ in my model.[7] Figure 5 illustrates impulse responses to a positive money growth shock with the non-

negativity constraint on the nominal interest rate under $\alpha = \delta = 0.7937$. The result implies that the rebound of the liquidity effect is extremely small. By reducing the cost parameters, α and δ, decreases of the nominal interest rate become small in the money demand equation. The zero lower bound tightens the economy by constraining the nominal interest rate, but its effect is quietly small because the decrease of the nominal interest is small to begin with. This empirically assessed result implies that it is not necessary for monetary authorities to pay attention to the contraction effect of the zero lower bound. As described in the previous subsection, since the tightening effect of the zero lower bound through the liquidity effect is not large under $\alpha = \delta = 2$, it might not necessarily be important to develop policies to mitigate the structures of the liquidity effect generated through the money demand equation.

4. Conclusion

In this paper, I quantitatively analyzed the tightening effect of the zero lower bound on the nominal interest rate. The main findings are as follows. (1) When the nominal interest rate hits the zero lower bound, the existence of a structure which generates the strong liquidity effect mitigates the effects of a money growth shock. (2) These results show that the tightening effect of the zero lower bound on a monetary policy shock might depend on the strength of the reduction in the nominal interest rate, i.e., the potential liquidity effect. (3) In my assessment of the empirical literature, however, the model implied that the tightening effect of the zero lower bound through the rebound of the liquidity effect is quite small. Thus, while the tightening effect of the zero lower bound is non-trivial in monetary policy, it may not be necessary to pay a great deal of attention to it in monetary policy.

NOTES

1. To say that the model does not generate the liquidity effect means that the model does not generate a sufficient reduction of the nominal interest rate toward a rise in money growth.
2. Christiano and Gust (1999) introduced the adjustment cost function to provide a persistent response of the liquidity effect. Andrés et al. (2004) and Nelson (2002) also used this cost function for a similar reason.
3. See Walsh (2010), Chapter 8 for details of the derivation.
4. Nelson (2002) chose $\alpha = \delta = 0.43$ considering an empirical assessment; however, since I am interested in the effects of the zero lower bound in the situation where a strong liquidity effect exists, I choose the parameter value $\alpha = \delta = 2$ following Christiano and Gust (1999). To consider the empirical assessment, model behavior with the zero lower bound under $\alpha = \delta = 0.7937$ is analyzed in subsection 3.3.
5. The higher the persistence in money growth becomes, the more positively the nominal interest rate tends to respond to the positive money growth.
6. See Guerrieri and Iacoviello (2014) for details about piecewise linear solutions.
7. Taylor (1993) and Janssen (1998) estimated the coefficient of the real money balance in the money demand equation for the U.S. and U.K., respectively. Nelson (2002) set $\psi = 10$

following these results. This setting is consistent with my model since the money demand equation is mostly identical (see Nelson 2002, p.700). Then, $\psi = 10$ implies $\alpha = \delta = 0.7937$ in my model.

ACKNOWLEGDMENTS

REFERENCES

Andrés, J., López-Salido, J. D & Nelson, E. (2004). Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking*, *36*(4), 665-690.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, *12*(3), 383-398.

Christiano, L. J. (1991). Modeling the liquidity effect of a money shock. *Quarterly Review*, Federal Reserve Bank of Minneapolis, *15*(1), 3-34.

Christiano, L. J., & Eichenbaum, M. (1995). Liquidity effects, monetary policy and the business cycle. *Journal of Money, Credit and Banking*, *27*, 1113-1136.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1997). Sticky prices and limited participation models: A comparison. *European Economic Review*, *41*, 1201-1249.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1998). Modeling money. NBER Working Paper 6371.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? Chapter 2, in J. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, Amsterdam: Elsevier Science Publication.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, *113*(1), 1-45.

Christiano, L. J., & Gust, C. J. (1999). Discussion of Andrew Levin, Volker Wieland, and John C. Williams, 'Are simple monetary policy rules robust to model uncertainty?' in J. Taylor (ed.), *Monetary Policy Rules*, Chicago: The University of Chicago Press.

Edge, R. M. (2007). Time-to-build, time-to-plan, habit-persistence, and the liquidity effect. *Journal of Monetary Economics*, *54*, 1644-1669.

Guerrieri, L., & Iacoviello, M. (2014). Occbin: A toolkit to solve models with occasionally binding constraints easily. working paper, Federal Reserve Board

Hasui, K. (2013). The non-negative constraint on the nominal interest rate and the effects of monetary policy. MPRA Paper 49394, University Library of Munich, Germany

Janssen, N., (1998). The demand for M0 in the United Kingdom reconsidered: some specification issues. Bank of England, Working Paper No. 83.

Keen, B. D. (2004). In search of the liquidity effect in a modern monetary model. *Journal of Monetary Economics*, *51*(7), 1467-1494.

Leeper, E. M., & Gordon, B. D. (1992). In search of the liquidity effect. *Journal of Monetary Economics*. *29*, 341-369.

Leeper, E. M., Sims, C. A., & Zha, T. (1996). What does monetary policy do? *Brookings Papers on Economic Activity*, *2*, 1-178.

Nelson, E. (2002). Direct effects of base money on aggregate demand: Theory and evidence. *Journal of Monetary Economics*, *49*(4), 687-708.

Taylor, J. B., (1993). *Macroeconomic Policy in a World Economy*. WW Norton, New York.

Walsh, C. E. (2010). *Monetary Theory and Policy*. (3rd ed) The MIT Press, Cambridge, Massachusetts.